PHYSNET

MISN-0-468

RELATIVISTIC KINEMATICS AND EINSTEINIAN OPTICS

Relativity

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

RELATIVISTIC KINEMATICS AND EINSTEINIAN OPTICS

by

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Title: **Relativistic Kinematics and Einsteinian Optics**

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Input Skills:

1. Vocabulary: Doppler effect, Fizeau drag effect, stellar aberration.
2. State the Lorentz transformation for a boost along any coordinate axis (MISN-0-466).

Output Skills (Knowledge):

K1. Derive the velocity and acceleration transformation equations for a standard Lorentz transformation.

K2. Derive the correct relativistic expressions for (a) the Fizeau drag effect, (b) the Doppler effect, (c) stellar aberration.

Output Skills (Rule Application):

R1. Given information about the velocity and acceleration (magnitudes and directions) of a body in one inertial frame, find the corresponding quantities in another frame.

R2. Given the wavelength of light a source emits at rest find its observed wavelength when it is in motion relative to some observer.

External Resources (Required):

1. W. Rindler, *Essential Relativity* Van Nostrand, (1977).
2. R. Resnick, *Basic Concepts in Relativity and Early Quantum Theory*, Wiley (1972).

External Resources (Optional):

1. Scott and Van Driel, "Geometrical Appearances at Relativistic Speeds," in *The American Journal of Physics*, 38, p. 971 (Aug. 1970).

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New authors, reviewers and field testers are welcome.

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1. Introduction

This unit completes the kinematics of special relativity and includes discussions of velocity and acceleration transformations as well as some of their consequences for optics.

2. Procedures

1. Read Rindler, section 2-15 . Fill in the details leading from eqs. 2.23, 2.24 and 2.8 to eq. 2-25. Recall the limit definition of a derivative.

   Also, fill in the details leading to eqs.2.27 and 2.28 in Rindler.

   Question: In MISN-0-466 you were asked in an exercise to show that

   $$\gamma = \gamma_1 \gamma_2 (1 + v_1 v_2)$$

   Compare that result with the first part of eq. 2.28 in Rindler. Why are they similar? In what way are they different?

   ▷ Exercise - Consider three inertial frames S, S' and S''. S' is boosted along x relative to S with speed v_1 while S'' is boosted along y relative to S' with speed v_2'. Show that, even though S'' is parallel to S', and S' is parallel to S, S'' is not parallel to S and that the x'' axis makes an angle $\Delta\phi$ with the x-axis where

   $$\tan \Delta\phi = -\gamma_1 v_1 v_2$$

   with v_1 and v_2 the x and y-components of the velocity of S'' relative to S.

   Comment - The preceding exercise is very closely related to Thomas precession which played an historic role in the understanding of electron spin effects in atomic spectra.

   (Optional) Read about Thomas precession in Eisberg, pp. 340-343. More exhaustive treatments are given in the book by C. Moller, *The Theory of Relativity* (Oxford Press, New York, 1952), pp. 53-56 and

   in the paper by George P. Fisher, The Thomas Precession, *American Journal of Physics 40*, 1772 (Dec. 1972). There is also a nice (but more difficult) paper by W. E. Furry, Lorentz Transformations and the Thomas Precession, which appeared (I believe) in the *Physical Review* in the late 1950's. A thorough discussion in terms of electron spin is given in the book by S. D. Sard, *Relativistic Mechanics* (Benjamin, 1970), pp. 285-290.

   ▷ Exercise - The acceleration of a particle is defined as usual by

   $$\vec{a} = \frac{d\vec{u}}{dt}$$

   ▷ Show that the acceleration transformation equations are:

   $$a_1' = \frac{a_1}{\gamma^3 (1 - v u_1)^3}$$

   $$a_2' = \frac{a_2}{\gamma^2 (1 - v u_1)^2} + \frac{u_2 v a_1}{\gamma^2 (1 - v u_1)^3}, \text{ etc.}$$

2. a. Read Rindler, section 3.1.

   b. Read French, pages 136-137. This is a nice example of the use of Minkowski diagrams. The result obtained here is called the longitudinal (or radial) Doppler effect.

   Read Rindler, section 3.2. The general Doppler effect formula is given in eq. 3.3. You should be able to derive this result in a lucid fashion. Note that the result involves two effects occurring concurrently: time dilation relating the time interval between crests as seen by the observer and as seen in the rest frame of the source, and time delay due to displacement of the source away from the observer.

   ▷ Exercise - Suppose that you are viewing through binoculars a clock that is moving away from you with a speed v. How much time elapses on your (rest) clock for each unit τ_0 of time that you observe the moving clock to register?

   ▷ Exercise - Show that the transverse Doppler effect (viewed at 90° w. r. t. the relative motion) is given by

   $$\nu = \frac{\nu_0}{\gamma}, \quad (\gamma = (1 - u^2)^{-1/2})$$

   where ν_0 is the rest frequency of the source (speed u) and ν is the observed frequency.

c. Read French, pp. 132-134 and Rindler, pp. 57-58. You should be able to derive Eqs. 41.1 and 41.2 in Rindler.

d. (Optional) There are two interesting theorems relating to the visual appearance of moving objects - the stereographic projection theorem and the supersnapshot theorem.

Stereographic projection theorem (Penrose) - Two observers in different inertial frames see images of an object which have the same stereographic projection (from unit spheres centered on the observers to planes tangent to the sphere and perpendicular to the line of relative motion) except for a scale factor of $\sqrt{(1-v)/(1+v)}$ where v is the relative speed of the two observers.

This theorem is based on eq. 3.9 on p. 58 of Rindler which can be obtained from eqs. 3.7 and 3.8. The theorem itself is discussed on pages 58 and 59 of Rindler. Additional discussion can be found in the paper by Scott and van Driel, Geometrical Appearances at Relativistic Speeds, *American Journal of Physics 38*, 971 (Aug. 1970).

Supersnapshot theorem (Terrell) - Two photons that travel abreast a fixed distance d apart in one inertial frame do so in *all* inertial frames.

Read Rindler, pp. 59-60.

3. ▷ Exercise - Frame S' is boosted along the $+x$-axis with speed v relative to frame S. A rigid rod at rest in the xy-plane of frame S makes an angle θ with the x-axis. A jet-propelled ant is sliding along the rod at a relativistic speed V in frame S.

a. What is the orientation of the rod in frame S'? (use Lorentz contraction)

b. What is the angle between the ant's velocity and the x'-axis in frame S'? (use velocity transformation)

c. Are these two results compatible? Explain.

▷ Work problems 5-2, 5-12, and 5-16 in French.

▷ Work problems 3.11 and 3.12 in Rindler.

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