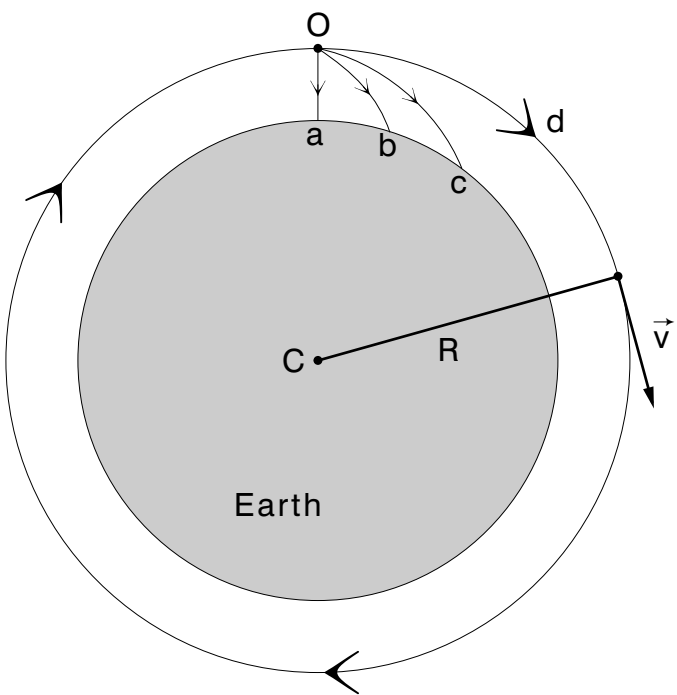


**CIRCULAR MOTION
WITH CONSTANT SPEED**



CIRCULAR MOTION WITH CONSTANT SPEED
by
F. Reif, G. Brackett and J. Larkin

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Input Skills:

1. Vocabulary: velocity, speed, acceleration (MISN-0-406).
2. Express velocity and acceleration as the time rate of change of appropriate vector quantities (MISN-0-406).
3. Describe the horizontal and vertical motion of a projectile near the earth's surface (MISN-0-407).

Output Skills (Knowledge):

- K1. Vocabulary: period, rotational frequency, centripetal acceleration.
- K2. For a particle moving in a circle with constant speed, derive the direction and state the magnitude of its acceleration.

Output Skills (Problem Solving):

- S1. For a particle moving in a circle with constant speed, use any given two of these quantities to calculate the third: acceleration, speed, and orbital radius.
- S2. For a particle moving with constant speed along a circular path of given radius, use the period or frequency to calculate its speed or acceleration.
- S3. Given a constant orbital radius of a satellite and its centripetal acceleration due to gravity, calculate the satellite's orbital speed and period.

Post-Options:

1. "The Gravitational Force" (MISN-0-410).

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F. Reif, G. Brackett and J. Larkin

1. Overview

There are many important situations where a particle moves in a circular path with constant speed. For example, the earth moves around the sun in a nearly circular orbit with a nearly constant speed. Another example is the centrifuge commonly used in chemistry and biology: as the centrifuge rotates, the sample placed near its rim moves around a circle with constant speed. * Indexacceleration, centripetal,

2. Centripetal Acceleration

2a. Acceleration in Constant-Speed Circular Motion. In Fig. 1 we exhibit the path of a particle that moves with constant speed v around a circle of radius r . We place the origin of the coordinate system at the center of the circle so the position vector \vec{r} of the particle will have a constant magnitude equal to the radius r of the circle. We indicate the velocity \vec{v} of the particle by an arrow drawn from the particle: this velocity has at any instant a direction that is along the circular path (i.e., perpendicular to \vec{r}). As the particle moves around the circle, the magnitude of its velocity (its speed) remains constant. However, its velocity changes continually because of its continual change in direction. Since its velocity is changing, its acceleration can not be zero. What is the direction and magnitude of this acceleration?

2b. A Diagram at Two Close Times. A diagram can be used to show the relationships between radius, velocity, and acceleration for a circularly orbiting particle. In Fig. 2a we show an instant of time t_0

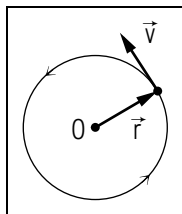


Figure 1. Motion of a particle with constant speed v around a circle of radius r .

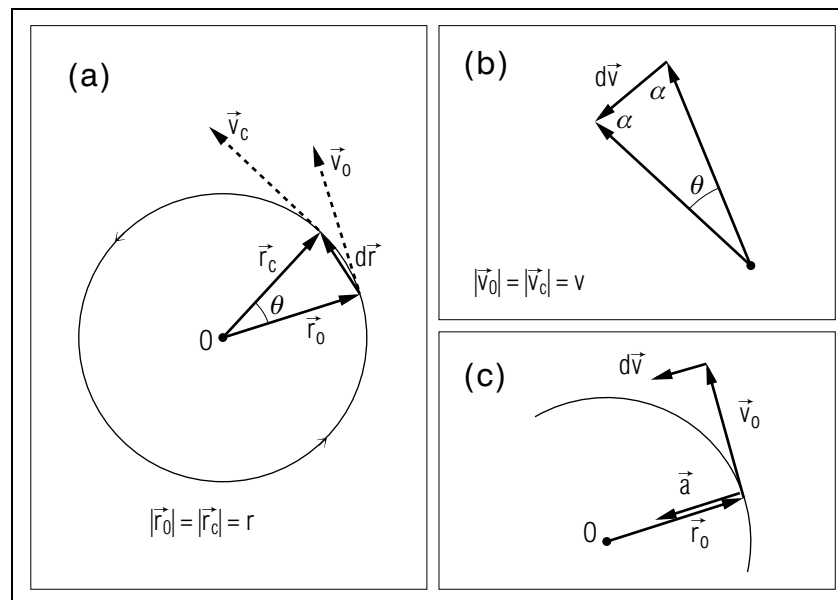


Figure 2. For a particle moving along a circular arc with constant speed: (a) position \vec{r} and velocity \vec{v} at two slightly different times; (b) velocity change $d\vec{v}$ between the two times; (c) relation between the directions of \vec{r}_0 , \vec{v}_0 , and acceleration \vec{a} . The magnitude of θ is exaggerated.

when the particle is located at a point \vec{r}_0 and is moving with a velocity \vec{v}_0 . At a slightly later time, t_c , the particle has moved to a nearby point labeled \vec{r}_c and it now has a velocity labeled \vec{v}_c . In Fig. 2a we also show $d\vec{r}$, the small change in the particle's position during the small time interval $dt = t_c - t_0$. In Fig. 2b we show the corresponding small change $d\vec{v}$ in the velocity. Such vector diagrams are sufficient to answer all questions about the particle's acceleration $\vec{a} = d\vec{v}/dt$.

2c. Direction of the Acceleration. Using Fig. 2 we can find the direction of the acceleration for an object following a circular path with a constant speed. First note that since the speed is constant the magnitudes of the velocities \vec{v}_0 and \vec{v}_c in Fig. 2a are equal. Then these two sides of the triangle in Fig. 2b are equal and so also are the two angles α . That makes $2\alpha + \theta = 180^\circ$ since the sum of all angles of a triangle is equal to 180° . But since we take the time interval dt to be very small, the velocity change $d\vec{v}$ is so small that the angle θ is very small compared to

the other angles. Hence 2α is very close to 180° so α is very close to 90° . This means that the small velocity change $d\vec{v}$ of the particle, and thus also the particle's acceleration, $\vec{a} = d\vec{v}/dt$, is very close to perpendicular to the velocity \vec{v}_0 . In fact, constant-speed acceleration must be exactly perpendicular to the velocity at all times: any component of acceleration in the direction of the velocity would cause the speed to be non-constant. Now \vec{v}_0 is itself perpendicular to \vec{r}_0 (see Fig. 2c), so \vec{a} must be parallel to \vec{r}_0 and, from the direction of $d\vec{v}$, it must be opposite in direction to \vec{r}_0 :

- When a particle moves in a circular path with constant speed, the direction of its acceleration at any instant is toward the center of the circle.

Such acceleration is called “centripetal,” meaning “directed toward the center.”

2d. Magnitude of the Acceleration. We now use Fig. 2 to find the magnitude of the acceleration for an object following a circular path with constant speed. First, note that the velocity \vec{v}_0 is perpendicular to \vec{r}_0 ; similarly, \vec{v}_c is perpendicular to \vec{r}_c . This means that the angle θ between \vec{v}_0 and \vec{v}_c in Fig. 2b is the same as the angle θ between \vec{r}_0 and \vec{r}_c in Fig. 2a. Furthermore, the triangle in Fig. 2b has two equal sides since $|\vec{v}_0| = |\vec{v}_c| = v$; similarly, the triangle in Fig. 2a has two equal sides since $|\vec{r}_0| = |\vec{r}_c| = r$. Hence the triangles in these two figures are geometrically similar. Because of the proportionality of corresponding sides of similar triangles, the ratio of the side opposite the angle θ to the side adjacent must be the same for each of the triangles. Symbolically:

$$\frac{|d\vec{v}|}{v} = \frac{|d\vec{r}|}{r}, \quad (1)$$

so that

$$|d\vec{v}| = \frac{v}{r} |d\vec{r}|. \quad (2)$$

Then the magnitude of the acceleration, $|\vec{a}|$, is:

$$|\vec{a}| = \frac{|d\vec{v}|}{|dt|} = \frac{v}{r} \frac{|d\vec{r}|}{|dt|} = \frac{v}{r} v, \quad (3)$$

since $|d\vec{r}|/|dt|$ is the speed v (the magnitude of the particle's velocity $\vec{v} = d\vec{r}/dt$). Then the magnitude of the acceleration, $a = |\vec{a}|$, is:

$$a = \frac{v^2}{r}. \quad (4)$$

The statement in Sect. 2c, plus Eq. (4), specify completely the acceleration of a particle moving around a circular path with constant speed.

3. Describing Rotation

3a. Period and Rotational Frequency. Whether an object is spinning or orbiting, its rate of rotation can be described in several different but equivalent ways depending on which is more convenient. For example, we may use the “period” of rotation, commonly denoted by T , which is defined as the time required for one revolution. Alternatively, we may use the “rotational frequency,” commonly denoted by ν ,¹ which is defined as the number of revolutions per unit time. Here the phrase “per unit time” means “divided by the corresponding time.” For example, an ultracentrifuge commonly used in biochemistry rotates through 60,000 revolutions every minute so its rotational frequency is:

$$\nu = \frac{(60,000)}{(60\text{ s})} = 1000/\text{s}.$$

The two descriptive concepts, period T and rotational frequency ν , are simple inverses:

$$\nu = T^{-1}. \quad (5)$$

This is because T is some time interval divided by the corresponding number of revolutions, while ν is some number of revolutions divided by the corresponding time interval.

3b. Rotational Speed. If a particle travels along a circular path at constant speed, that speed can be calculated from either the particle's period or its rotational frequency. The distance traveled by such a particle increases proportionately to the elapsed time and the particle traverses one circumference, $2\pi r$, in the time T needed for one revolution. Thus the particle's speed v is:

$$v = \frac{2\pi r}{T} \quad \text{or} \quad v = 2\pi r\nu. \quad (6)$$

Equation (6) can be used to calculate the speed of the particle from either its period or its rotational frequency. That speed can then be used in Eq. (4) to calculate the magnitude of the particle's acceleration. Equation (6) is valid not only for a single particle traversing a circular orbit, but also for a particle that is part of an extended object. If this extended

¹The Greek letter ν (“nu”).

object is rotating about some axis at a constant rate, then every particle in the object travels with constant speed around its own circle of radius equal to the distance of that particle from the axis of rotation, so that speed can be found from the particle's radius and the object's period or rotational frequency.

4. Satellite Motion

4a. A Particular Case of Projectile Motion. The circular motion of a particle around a large massive body may be understood by treating the system as a special case of projectile motion.² Figure 3 illustrates what happens to a particle located near the surface of the earth when it is projected with successively larger horizontal components of velocity. The vertically downward gravitational acceleration of the particle is always directed toward the center of the earth, while the horizontal component of velocity is parallel to the surface of the earth. As the horizontal velocity of the particle is made larger, the particle travels a larger distance before it strikes the earth. Thus we can imagine projecting the particle with a horizontal component of velocity just large enough so that the particle never strikes the surface of the earth at all, but travels forever parallel to the earth's surface. In that case the particle simply travels around the earth in a circular path like that labeled (d) in Fig. 3. Indeed, such satellites of the earth have become familiar objects in recent years.

4b. Centripetal Acceleration of a Satellite. An earth satellite, like any other particle when it is subject only to the gravitational interaction with the earth, moves so that its centripetal acceleration is:

$$\vec{a} = \vec{g} \quad (\text{satellite}), \quad (7)$$

where the gravitational acceleration is independent of all properties of the satellite. Suppose that the satellite moves with a constant speed v in a circular orbit of radius R . Then the direction of \vec{a} is toward the center of this orbit, i.e., toward the center of the earth (see Sect. 2c). Thus \vec{a} indeed has the same direction as \vec{g} , as required by Eq. (7). The only additional requirement of Eq. (7) is that the magnitude of \vec{a} and \vec{g} must be equal:

$$a = g \quad (\text{satellite}). \quad (8)$$

²See "Component Description of Vectors and Motion" (MISN-0-407), especially Fig. E-2.

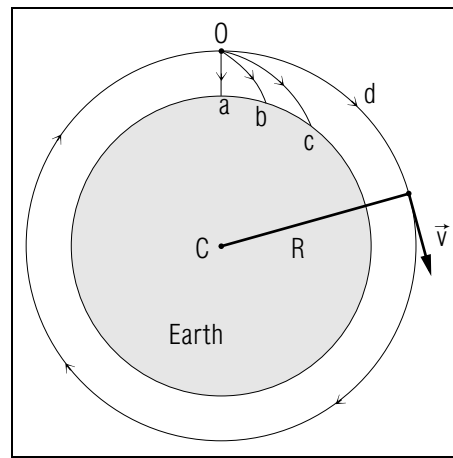


Figure 3. Motion of a particle projected with various horizontal velocities from a point near the surface of the earth: (a) particle falling vertically; (b) particle projected with a small horizontal velocity; (c) particle projected with a larger horizontal velocity; (d) particle projected so that it travels in a circular orbit around the earth.

4c. Orbital Speed and Period of a Satellite. We will now use Eqs. (4) and (8) to find the orbital speed of a satellite in terms of the satellite's orbital radius and the value of g at the satellite's distance from the earth's center.³ We already know from Eq. (4) that the magnitude of the satellite's centripetal acceleration is $a = v^2/R$, and we know from Eq. (8) that a satellite's acceleration is that of gravity. Combining these two we get:

$$\frac{v^2}{R} = g. \quad (9)$$

Hence:

$$v^2 = gR \quad \text{so} \quad v = \sqrt{gR}. \quad (10)$$

If the satellite moves in an orbit close to the surface of the earth, g has approximately the value 10 m/s^2 . If the satellite moves in an orbit far from the surface of the earth, the value of g is smaller. The moon is such a satellite moving in an orbit of very large radius.

5. Solving Problems: Advice

Applying Eqs. (4)-(6) for a solution that is clear and easy to check:

1. Write down the symbol corresponding to the desired quantity, the "end result" or "answer" for the problem at hand.

³See Problem 12 in the Problem Supplement.

- List the symbols corresponding to the quantities whose numerical values are given in the problem statement.
- Use one or more of Eqs. (4)-(6) to find an algebraic expression for the desired quantity in terms of the symbols with known values.
- Substitute the known values for the corresponding symbols, including units. If you wish to change units, take a unit name in the expression and replace it with the equivalent number of desired units (e.g. replace the word “foot” with “12 inches”).
- Rewrite the expression, grouping separately the units, the powers of 10, and the numerical factors, where possible.
- Simplify each of the three groups described in Step 5.
- Check that the resulting units are correct, and that the resulting magnitudes and directions of answers seem reasonable.

For a worked example using these steps, see [S-1]. Otherwise, wait until you get to Problems 4 and 5 in the Problem Supplement and see if you can use the steps appropriately.

Acknowledgments

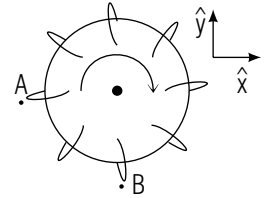
Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Glossary

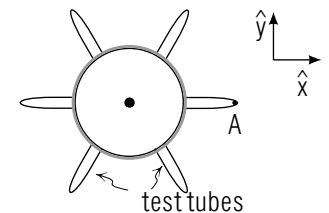
- centripetal acceleration:** the radially-inward acceleration of a particle moving in a circular path. This “center seeking” acceleration is present even when the particle has a constant speed.
- rotational frequency:** the number of rotations or revolutions per unit time made by an object undergoing circular motion; usually represented by the Greek letter ν (“nu”). Frequency is the inverse of period.
- period:** the time required for an object undergoing circular motion to make a complete rotation or revolution; often represented by the letter T . Period is the inverse of frequency.

PROBLEM SUPPLEMENT

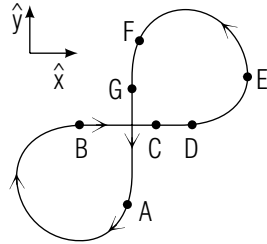
- A playground merry-go-round consists of a flat wooden platform which children can rotate by using handles at the edge (see figure). A child sits at the edge of a merry-go-round of radius 1.4 meter, which turns as indicated in the figure so that the child has a constant speed of 1.2 m/s.



- What are the child’s acceleration \vec{a} and velocity \vec{v} when he is at point A and when he is at point B?
 - Suppose the child moves with the same speed, but the merry-go-round turns in the opposite direction. What now are the child’s acceleration and velocity at point A?
 - Does changing the direction of motion change a particle’s velocity at a given point? Does it change the particle’s acceleration at that point? *Help:* [S-2]
- A car travels with the same constant speed first along a “gentle curve” (a circular path which has a large radius) and then along a “tight curve” (a circular path with a small radius). For which curve is the magnitude of the car’s acceleration larger?
 - Two cars travel along the same circular curved section of road, but, the speed of car A is 3 times as large as the speed of car B. Compare the accelerations of these two cars by expressing the magnitude of the acceleration of car A as a number times the magnitude of the acceleration of car B. *Help:* [S-5]
 - A laboratory centrifuge produces an acceleration $\vec{a} = (-1.6 \times 10^4 \text{ m/s}^2)\hat{x}$ for a sample at the point A on its rim (see figure). If the sample is 0.1 m from the center of the centrifuge, and moves with constant speed along a circular path, what is the speed of this sample? *Help:* [S-4]



4. The rotation of the earth has a period of 1 day = 9×10^4 sec. During this period an object on the earth's equator travels (relative to the earth's center) once around a circle of radius 6×10^6 m (the earth's radius). What is the magnitude of the centripetal acceleration of a palm tree at the equator? Compare this acceleration with the acceleration g of gravity by finding the ratio a/g . *Help: [S-1]*
5. An ultracentrifuge used in biological studies produces exceedingly large accelerations on samples located a distance 0.07 meter from its center by rotating them with a frequency $\nu = 6 \times 10^4/\text{min}$. What is the magnitude of such a sample's centripetal acceleration in terms of the unit m/s^2 ? Compare this acceleration with the acceleration g of gravity by finding the ratio a/g . *Help: [S-1]*
6. a. Suppose a driver travels around a curve of radius 225 meters with a safe speed v_0 (i.e., a speed corresponding to a safe acceleration \vec{a}_0 for which the wheels do not skid). He then approaches a curve of radius 100 meters. To maintain his safe acceleration of magnitude a_0 , should he increase, decrease, or maintain the same speed?
- b. If the driver maintains this safe acceleration, express the safe speed v' for the second curve in terms of the safe speed v_0 for the first curve. *Help: [S-3]*
7. The flat horizontal track of a toy railroad (see figure) consists of two circular segments of radius 0.1 m (between A and B and between D and F) joined by straight segments. A toy engine travels with uniformly increasing speed from A to D , and then travels from D to A with a constant speed of 0.2 m/s.



- a. Use the unit vectors \hat{x} and \hat{y} to describe the direction of the engine's velocity and acceleration at the points C and E .
- b. Find values for the engine's velocity and acceleration at the points E and G .
- c. Does the relation ($\vec{a} = v^2/r$, towards the center) correctly describe the engine's motion as it travels between A and B ? Does this relation describe the motion between F and A ?
8. If a car's acceleration has too large a magnitude, its wheels may skid. For ordinary cars, a safe acceleration has a magnitude no larger

- than one half the magnitude g of the gravitational acceleration at the earth's surface. For a circular section of a cloverleaf intersection of radius 125 m, what is the maximum safe speed corresponding to this acceleration? Express your answer by using the unit mi/hr (miles per hour) where $1 \text{ m/s} = 2.2 \text{ mi/hr}$.
9. In designing a centrifuge, it is often important that the centripetal acceleration of a sample placed at its rim be as large as possible.
- a. For a centrifuge with a given radius, should the sample be rotated faster or slower to increase the magnitude of its acceleration?
- b. Consider two centrifuges (1 and 2) in which samples travel in circles with different radii r_1 and r_2 , but with the same speed v . If r_1 is smaller than r_2 , which centrifuge provides the larger magnitude of acceleration for the sample at its rim?
- c. If $r_2 = 2r_1$, is the magnitude a_1 of the acceleration produced by centrifuge 1 twice as large, four times as large, half as large, or one-fourth as large as the magnitude a_2 of the acceleration produced by centrifuge 2?
10. A long-playing record on a turntable rotates with a frequency of about 30/min (more exactly 33.3/min). What is the magnitude of the centripetal acceleration of a dust particle located 0.15 meter from the record's center? What is the speed v of this particle? (Express your answers in terms of the units meter and second).
11. A satellite moving near the earth's surface travels once around its orbit in 84 minutes or in about 5.0×10^3 seconds. The radius of this orbit is very nearly the radius of the earth, 6.4×10^6 meter. What is the magnitude of such a satellite's centripetal acceleration? Check your result by comparing it with the gravitational acceleration at the earth's surface, $g = 10 \text{ m/s}^2$.
12. Many artificial satellites move very near the earth's surface (e.g., at a height of 300 km above the earth's surface, a distance much smaller than the earth's radius). Such a satellite then has a circular orbit of radius approximately equal to the earth's radius, $R = 6.4 \times 10^6$ m, and its centripetal acceleration has very nearly the magnitude $g = 10 \text{ m/s}^2$.
- a. Find the orbital speed of such a satellite.
- b. What is the orbital period of this satellite? Express T in units of minutes.

Brief Answers:

1. a. At A: $\vec{a} = 1.0 \text{ m/s}^2 \hat{x}$, $\vec{v} = 1.2 \text{ m/s } \hat{y}$.
At B: $\vec{a} = 1.0 \text{ m/s}^2 \hat{y}$, $\vec{v} = -1.2 \text{ m/s } \hat{x}$.
- b. $1.0 \text{ m/s}^2 \hat{x}$, $-1.2 \text{ m/s } \hat{y}$
- c. yes, no
2. a. Larger for the tight curve
- b. $a_A = 9a_B$
3. 40 m/s
4. $a = 3 \times 10^{-2} \text{ m/s}^2$, $a/g = 3 \times 10^{-3}$
5. $a = 3 \times 10^6 \text{ m/s}^2$, $a/g = 3 \times 10^5$
6. a. decrease
- b. $v' = 0.667v_0$
7. a. at C: velocity and acceleration along \hat{x} ;
at E: velocity along \hat{y} , acceleration opposite to \hat{x} .
- b. at E: $0.2 \text{ m/s } \hat{y}$, $-0.4 \text{ m/s}^2 \hat{x}$;
at G: $-0.2 \text{ m/s } \hat{y}$, 0.
- c. from A to B: No, since the speed v is not constant.
from F to A: No, unless you regard a straight path as a circular path with an infinitely large radius r . Then $a = v^2/r = 0$ correctly describes the motion between F and A.
8. 55 mi/hr
9. (a) faster; (b) centrifuge #1; (c) twice as large.
10. $a = 1.5 \text{ m/s}^2$, $v = 0.47 \text{ m/s}$
11. $a = 10 \text{ m/s}^2$, as expected for an object moving due to gravitational interaction near the earth's surface.
12. (a) $v = 8.0 \times 10^3 \text{ m/s}$;
(b) $T = 5.0 \times 10^3 \text{ s} = 83 \text{ min}$

SPECIAL ASSISTANCE SUPPLEMENT

S-1

(from TX-5, PS-Problems 4,5)

The earth travels in a circular path of radius 1.5×10^{11} meters around the sun, completing one revolution in a period of one year. What is the magnitude of the earth's centripetal acceleration (relative to the sun)? Express the answer in terms of the unit m/s^2 , using the conversion factor 1 year = 3.14×10^7 seconds.

Solution:

We can use the relation $a = v^2/R$, if we express the unknown speed v in terms of known quantities:

$$v = \frac{2\pi R}{T} \implies v^2 = \frac{4\pi^2 R^2}{T^2},$$

so

$$a = \frac{v^2}{R} = \frac{4\pi^2 R^2}{T^2 R} = \frac{4\pi^2 R}{T^2}.$$

This expresses the desired quantity in terms of symbols with known values. Next we substitute values, including units, and replace the unit year by 3.14×10^7 seconds.

$$a = \frac{4(3.14)^2(1.5 \times 10^{11} \text{ m})}{(3.14 \times 10^7 \text{ s})^2} = 6.0 \times 10^{-3} \text{ m/s}^2.$$

The resulting unit is correct, and the magnitude (much smaller than the acceleration $g = 10 \text{ m/s}^2$ of gravity) isn't unreasonably large.

S-2

(from PS-Problem 1)

A particle's velocity is always along its path, i.e., it has the same direction as a very small displacement from the position of the particle along the path. A particle's centripetal acceleration is always directed from the particle's position toward the inside of any curved path. In particular, if the particle moves with constant speed along a circular path, its centripetal acceleration is directed towards the center of the circle.

S-3 (from PS-Problem 6)

Since $a = v^2/r$, if the radius r of the road curve decreases, the driver must correspondingly decrease his speed v to maintain a centripetal acceleration of constant magnitude. In this problem, the radius decreases by a factor of 100/225. If a is to remain constant, the driver must then decrease his speed by the square root of 100/225.

S-4 (from PS-Problem 3)

The relation $a = v^2/r$ is a relation between magnitudes. Be sure you use the magnitude a (which is a positive number), not the vector \vec{a} .

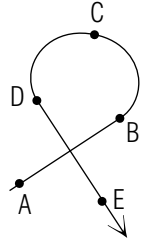
S-5 (from PS-Problem 2)

We want to use the relation $a = v^2/r$ to describe the dependence of a on v or on r , i.e., to use a comparison of values for v or r to compare corresponding values of a .

- Part (a): The speed v remains constant, but the radius r changes from a larger to a smaller value. Will a smaller value for r result in a larger or smaller value for $a = v^2/r$? Choose a value for v , e.g. $v = 10$ m/s. Then choose a smaller and a larger value for r , e.g. 50 m and 100 m. Find the value of a corresponding to each value of r .
- Part (b): The radius r is the same for both cars, but the speed v is 3 times as large for car A as for car B . Again choose values, for example $r = 100$ m and $v = 30$ m/s for car A and $v = 10$ m/s for car B . Then find the corresponding value of a for each car.

MODEL EXAM

1. See Output Skills K1-K2.
2. A small motor-powered boat follows the path shown in the drawing at the right. The path consists of a circular section between B and D , joining two straight sections. The boat travels with uniformly increasing speed from A to C and with constant speed from C to E . For each of the following pairs of points, write $\Delta\vec{v} = \vec{a}(\Delta t)$, $a = v^2/r$, neither, or both to indicate the relation(s) which describe the boat's motion as it moves between those points:



- a. A to B
 - b. B to C
 - c. C to D
3. A wooden camel is fastened a distance of 8 meters from the center of a merry-go-round platform which rotates with a period of 10 seconds. What is the centripetal acceleration of a child sitting on the camel?

Brief Answers:

1. See this module's *text*.
2. a. $\Delta\vec{v} = \vec{a}(\Delta t)$
b. neither
c. $a = v^2/r$
3. $\vec{a} = (3 \text{ m/s}^2, \text{ toward the center of the merry-go-round}).$

