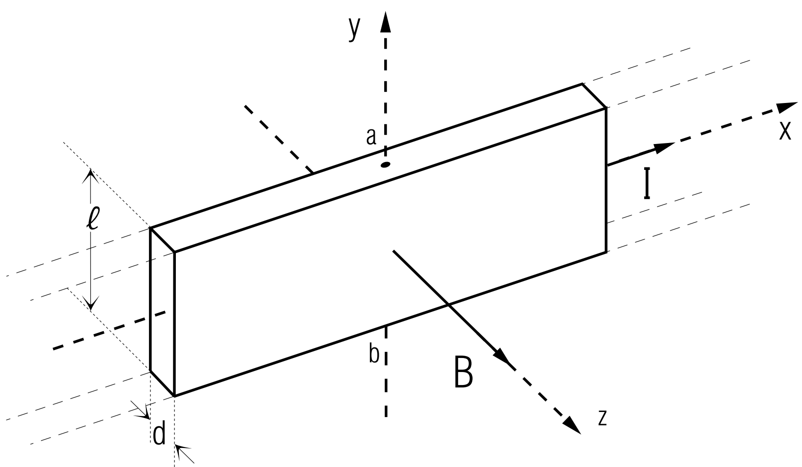
MISN-0-149

EXAMINING THE CHARGE CARRIERS; THE HALL EFFECT



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

EXAMINING THE CHARGE CARRIERS; THE HALL EFFECT

by

Peter Signell

Title: **Examining the Charge Carriers; The Hall Effect**

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Input Skills:

1. State the expression for the force (Lorentz Force) on a charged particle in electric and magnetic fields (MISN-0-122).
2. State the expression relating a set of charges and their common velocity to their equivalent value as a current (MISN-0-123).
3. Vocabulary: conduction in metals (MISN-0-118).

Output Skills (Knowledge):

K1. Starting from the Lorentz force, derive the equations which give the drift velocity and density of the microscopic charges constituting an electrical current in terms of the observable macroscopic Hall effect quantities.

K2. Outline briefly how one deduces from the Hall effect that the charge carriers in copper and silver are essentially the valence electrons of the constituent atoms.

Output Skills (Rule Application):

R1. Use the (given) measured value of the Hall constant to calculate the drift velocity of the charges in a specified current in a specified conductor.

External Resources (Optional):

1. C. M. Hurd, *The Hall Effect in Metals and Alloys*, Plenum Press (1972). For access, see this module's *Local Guide*.
2. F. Bitter, *Currents, Fields, and Particles*, Wiley (1957). For access, see this module's *Local Guide*.
3. C. Kittel, *Introduction to Solid State Physics*, John Wiley and Sons, New York (1986), p. 215. For access, see this module's *Local Guide*.

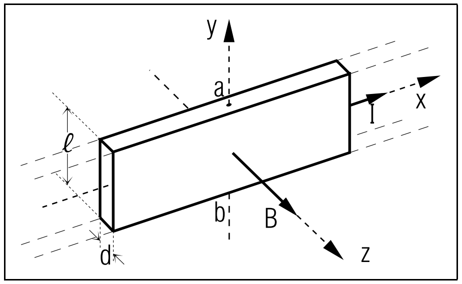

Figure 1. Section of a conductor through which is passing a current $\vec{I}$.

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1. Introduction

1a. Why We Study the Hall Effect. The moving objects constituting an ordinary electrical current are said by physicists to be negatively charged electrons, not positive charges as assumed in the electrical engineering convention.[1] These electrons' drift velocity down a household wire are said by physicists to be typically about 10 feet per hour. How can 10 ft/hr really be true when a wall switch seems to activate a lamp across the room instantly! Here we examine convincing evidence of the negative sign of the charge carriers and the magnitude of their drift velocity. Elsewhere we examine the speed of electrical power transmission.[2] The Hall effect is also interesting for its applications: it is used to determine electronic properties of new materials and for routine measurements of unknown magnetic fields.

1b. Mutually Perpendicular I, B, V. The term "Hall effect" refers to a special voltage that appears when a transverse magnetic field is applied to an electrical current flowing in a material (see Fig. 1). This special voltage, called the "Hall voltage," is at right angles to both the current and the magnetic field. This means that all three quantities involved, current, field, and voltage, are at right angles to each other. In the configuration shown in Fig. 1 the Hall voltage is measured along the y-axis,

[1]For further information see "Conductivity and Resistance" (MISN-0-118) and "Force on a Current in a Magnetic Field" (MISN-0-123).

[2]For further information see "Signal Velocity in a Conductor" (MISN-0-150).

with the voltmeter's positive lead at point b, negative lead at point a. Notice the orientations of the other two quantities.

1c. An $\vec{E}$ Field so a Lorentz Force. The existence of the y-axis voltage in Fig. 1 means that there is a y-axis electric field, one which, like all electric fields, exerts a y-axis force on charge carriers like those in the current I. It is not surprising that such a force is exerted on the charge carriers; it is just the ordinary Lorentz force that occurs whenever there is a magnetic field at right angles to the velocity of the charge. The importance of the effect is that the value and sign of the voltage tell us much about the charge carriers if we know the magnetic field. Then, the measurement having been made for a particular material in a known magnetic field, that material's Hall voltage is used industrially to determine unknown magnetic fields.

2. Delineating the Conditions

2a. Measurement Layout. The Hall voltage is measured across a piece of conducting material formed in the shape of a rectangular bar as in Fig. 1. We induce the current $\vec{I}$ in the x-direction and apply a constant magnetic field $\vec{B}$ in the z-direction.

2b. Force Independent of Charge Sign. If each charge carrier in the current $\vec{I}$ has charge q, then the magnetic field Lorentz force on the charge is:[3] $\vec{F} = q\vec{v}_D \times \vec{B}$, where $\vec{v}_D$is the velocity of drift in the direction of the current.

▷ Show that if the current consists of positive particles going to the right (positive x-direction) in Fig. 1 then the force on them is downwards (negative y-direction). Then make the other possible assumption that the current to the right consists of negative particles going to the left and show that such negative charge carriers for the same current would also experience a downward magnetic field Lorentz force.

2c. Downward Drift Reaches Equilibrium. When the current of Fig. 1 is initiated, the Lorentz Force causes a net downward migration of the charge-carrier electrons. This downward migration produces an increasing concentration of electrons in the lower part of the material, leaving a correspondingly increasing concentration of positive lattice ions in the upper part of the material. A carrier that is part-way down will thus

[3]See "Force on a Charge Particle in a Magnetic Field: The Lorentz Force" (MISN-0-122).

experience a net upward electrostatic field just due to the gradient in the net charge concentration. Downward migration ceases when the number of "lower" electrons becomes so large that the upward "concentration gradient" force matches the downward Lorentz force.

3. The Equilibrium Equations

3a. Equilibrium E and v. The condition for equilibrium in downward migration is that the net force on a carrier is zero. Writing the charge on the carrier as q, the condition is:

$$F_y = 0 = q[E_y + (\vec{v}_D \times \vec{B})_y].$$

Then each mobile charge moves in the x-direction in an electric field whose y-component is:[4]

$$E_y = +v_D\, B. \tag{1}$$

▷ Show that E_y is in the upward direction for positively charged carriers, downward for negatively charged carriers, and that this checks with the electric field direction being away from positive charges and toward negative charges.

3b. Equilibrium Voltage. The potential difference between points a and b of Fig. 1 is measurable with a voltmeter, and is simply related to E_y. Recall that the voltage of point b with respect to point a is the work per unit charge which you would have to do in moving a charge from the reference point a to point b. Then the Hall voltage is:

$$V_{ba} = -\int_a^b \vec{E}\cdot d\vec{y} = -\int_a^b (-E_y\, dy) = E_y \int_a^b dy = E_y\, \ell \text{ (see Fig. 1)},$$

where we have used the fact that E_y in Eq. (1) has no dependence on y, hence is constant in the y-direction. Since $\vec{E}$ is independent of y we could also have obtained V_{ba} by:

[4] This analysis has assumed that all the charge carriers have the same sign. If they don't, then V_D in (1) is an average (see Appendix).

$$\begin{aligned} V_{ba} &= \frac{\text{Work}(a \to b)}{\text{charge}} \\ &= \frac{\text{Applied Force}(a \to b) \times \text{Distance}(a \to b)}{\text{charge}} \\ &= \frac{\text{Applied Force}(a \to b)}{\text{charge}} \times \text{Distance}(a \to b) \\ &= E_y\, \ell \end{aligned} \tag{2}$$

where we have used the fact that the positive downward applied force per unit charge must exactly cancel the positive upward electric field E_y.

3c. Carrier Velocity. Combining Eqs. (1) and (2), we can eliminate E_y in terms of measurable quantities:

$$v_D = \frac{V_{ba}}{B\ell}. \tag{3}$$

Measurement of the Hall voltage thus gives us a direct measurement of the drift velocity of the charge carriers.

3d. Carrier Density; the Hall Constant. We can also obtain the sign of the charge and the density of the carriers by recalling that the current is given by:

$$\vec{I} = \frac{Q}{L}\vec{v}_D$$

where (Q/L) is the charge per unit length along the conductor. This relation can also be written as:

$$\vec{I} = \sigma_v\, A\vec{v}_D, \tag{4}$$

where σ_v is the carrier charge per unit volume, and A is the cross sectional area of the conductor. Combining Equations (3) and (4) and using $A = \ell d$ (see Fig. 1):

$$\sigma_v = \frac{IB}{V_{ba} d}. \tag{5}$$

The inverse of this quantity is called the "Hall constant" and is written R_H or C_H.

3e. Actual Measurements. When one actually makes the measurement on, say, a piece of copper which could have been used for ordinary house wiring, the Hall voltage turns out to be negative. This means that the voltmeter will only give a positive reading if the leads are reversed so that the (+) one is at point a. The inescapable conclusion is that these carriers have negative charge. The numerical value obtained in Eq. (5) turns out to be very close to the density of valence-electron-charge for both of the common good conductors, silver and copper. That value, the valence-electron charge per unit volume, can be obtained by dividing the metal's mass density by its mass per atom to get the number of atoms per unit volume and then multiplying that value by the valence-electron charge on each atom.[5]

Acknowledgments

Michael Harrison provided a helpful discussion of the Hall effect. William Lane, Stephen Smith and their students, especially Jim Peterson, provided much valuable feedback on earlier versions. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

A. Hall Constant for Two Carriers

(for those interested)

When a current consists of electrons (negative charge) and oppositely-moving electron holes (positive charge) as in iron, magnesium and semiconductors, the Hall constant is given by:[6]

$$R_H = \frac{p - nb^2}{|e|(p + nb)^2}$$

where p and n are the number densities of positive and negative carriers, respectively, $|e|$ is the magnitude of the electronic charge, and b is the "mobility ratio," $(\tau_m m_p)/(\tau_p m_n)$, where the τ's are the carriers' mean-free-times between collisions and the m's are their (positive) masses. The above formula for R_H is said to be in the "drift velocity approximation."

[5]See *Currents, Fields, and Particles*, F. Bitter, John Wiley and Sons, New York (1957), Table 6.1, p.231, and *The Hall Effect in Metals and Alloys*, C. M. Hurd, Plenum Press, New York (1972), Fig. 1.1 on page 7, and Table 7.6 on pages 278-9. For access, see this module's *Local Guide*.

[6]For further information see *Introduction to Solid State Physics*, C. Kittel, John Wiley and Sons, New York (1986), p. 215. For access, see this module's *Local Guide*.

As an example, R_H is positive for iron, showing a dominance of holes over electrons.

LOCAL GUIDE

The readings for this unit are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them as "The readings for CBI Unit 149." Do **not** ask for them by book title.

PROBLEM SUPPLEMENT

Note: Problem 3 also occurs in this module's *Model Exam*.

1. A Hall probe uses the Hall Effect to measure magnetic field strength. If the probe is copper, has a thickness of 0.1 mm, an R_H value of $R_H = -5 \times 10^{-11}\,\mathrm{m^3/(A\,s)}$, and can measure a potential of 0.1 mV, what current is needed to measure a field of 0.2 Tesla? Would you want to use copper for a Hall probe?

2. Using $R_H = -5 \times 10^{-11}\,\mathrm{m^3/(A\,s)}$, what is the charge carrier drift velocity for a 2.0 A current in a copper conductor whose cross-sectional area is (1
unitcm^2)? Why does the result imply the charge carriers are negative?

3. In Hurd's book, pages 278-9, the experimental values of R_H for copper cover the range:

$$-7.8 \times 10^{-11}\,\mathrm{m^3/(A\,s)} \le R_H \le -4.9 \times 10^{-11}\,\mathrm{m^3/(A\,s)}$$

Calculate the drift velocity (in feet per hour) for a 5 amp current in No. 18 copper wire (cross sectional area $2\,\mathrm{mm^2}$). Note: m/s = 1.18×10^4 ft/hr.

Brief Answers:

1. $R_H = \dfrac{V_{ba} d}{IB}$

so: $I = \dfrac{V_{ba} d}{RB} = \dfrac{(10^{-4}\,\mathrm{V})(10^{-4}\,\mathrm{m})}{(-5 \times 10^{-11}\,\mathrm{m^3/(A\,s)})(2.0 \times 10^{-1}\,\mathrm{T})} = -10^3\,\mathrm{A}.$

No: vastly too much current.

2. $v_D = \dfrac{V_{ba}}{B\ell} = \dfrac{R_H IB/d}{B\ell} = \dfrac{R_H I}{\ell d}$

$= \dfrac{(-5 \times 10^{-11}\,\mathrm{m^3/(A\,s)})(2.0\,\mathrm{A})}{(10^{-2}\,\mathrm{m})(10^{-2}\,\mathrm{m})} = -10^{-6}\,\mathrm{m/s}.$

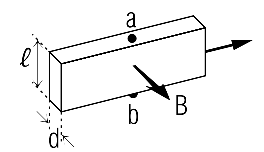
Here v_D is negative so it is opposite to the positive charge direction, so these charges are negative.

3. Combine the equations above to find v_D in terms of R_H, I and A.

   Then: $-2.3\,\text{ft/hr} \le v_D \le -1.4\,\text{ft/hr}$.

MODEL EXAM

1.



Starting from the Lorentz force, derive the Hall effect equations:

$$v_D = \frac{V_{ba}}{B\ell}$$

$$\sigma_v = \frac{IB}{V_{ba}d}$$

2. See Output Skill K2 in this module's *ID Sheet*.

3. In Hurd's book, pages 278-9, the experimental values of R_H for copper cover the range:

$$-7.8 \times 10^{-11}\,\text{m}^3/\text{A s} \le R_H \le -4.9 \times 10^{-11}\,\text{m}^3/\text{A s}$$

   Calculate the drift velocity (in feet per hour) for a 5 amp current in No. 18 copper wire (cross sectional area $2\,\text{mm}^2$). Note: $\text{m/s} = 1.18 \times 10^4\,\text{ft/hr}$.

Brief Answers:

1. See this module's *text*.

2. See this module's *text*.

3. See this module's *Problem Supplement*, problem 3.