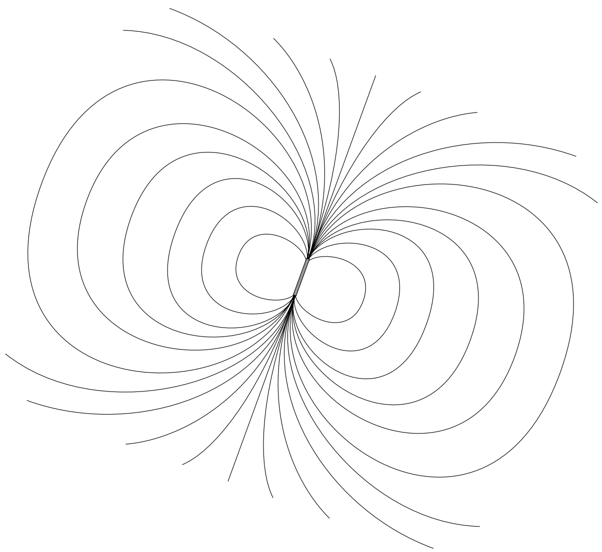
MISN-0-130

MAGNETIC DIPOLES



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

MAGNETIC DIPOLES

by
Kirby Morgan

Title: **Magnetic Dipoles**

Author: Kirby Morgan, HandiComputing, 319 E. Henry, Charlotte, MI 48813

Version: 11/6/2001 Evaluation: Stage 0

Length: 1 hr; 20 pages

Input Skills:

1. Vocabulary: electric dipole, electric dipole moment, point dipole (MISN-0-120).
2. Calculate the torque, potential energy, and work for an electric dipole in an external electric field (MISN-0-120).
3. Calculate the torque on a current loop in an external magnetic field (MISN-0-123).

Output Skills (Knowledge):

K1. Define the magnetic dipole moment (vector) for a system of N (fictitious) magnetic monopoles.

K2. Define the magnetic dipole moment for a magnetic dipole.

K3. Explain the existence of magnetic dipoles in spite of the apparent non-existence of magnetic monopoles.

K4. Write expressions for the torque, work, potential energy, and field of magnetic dipoles.

Output Skills (Problem Solving):

S1. Determine the torque on a magnetic dipole in an external magnetic field and the work done on it in changing its orientation.

S2. Determine the potential energy of a given magnetic dipole or system of dipoles in an external field.

S3. Determine the magnetic field at some given point in space due to a given magnetic dipole.

Post-Options:

1. "The Search for Magnetic Monopoles" (MISN-0-140).
2. "Magnetic Fields in Bulk Matter: Magnets" (MISN-0-141).

THIS IS A DEVELOPMENTAL-STAGE PUBLICATION
OF PROJECT PHYSNET

The goal of our project is to assist a network of educators and scientists in transferring physics from one person to another. We support manuscript processing and distribution, along with communication and information systems. We also work with employers to identify basic scientific skills as well as physics topics that are needed in science and technology. A number of our publications are aimed at assisting users in acquiring such skills.

Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

MAGNETIC DIPOLES

by

Kirby Morgan

1. Introduction

The magnetic dipole moment is defined in a manner similar to the electric dipole moment. This analogy can be made even though the magnetic monopole has never been observed in an experiment.

2. Fundementals of Magnetic Dipoles

2a. Defining the Magnetic Dipole Moment. The magnetic dipole moment can be defined in a manner analogous to that for the electric dipole moment.[1][2] The magnetic quantity that corresponds to an electric charge is the *magnetic monopole* (or, for short, *magnetic pole*).[3] Although a magnetic monopole has never been observed,[4] it nevertheless is used to define the general expression for the magnetic dipole moment.

Consider a collection of N magnetic monopoles, m_1, m_2, $\ldots \mathrm{m}_N$. Relative to a fixed coordinate system, each pole is located at a point given by a vector $\vec{r}$; $\vec{r}_1$ for m_1, $\vec{r}_2$ for m_2, etc. (see Fig. 1). The *magnetic dipole moment* $\vec{\mu}$ (or, for short, the *magnetic moment*) is a vector defined as:

$$\vec{\mu} \equiv m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots + m_N\vec{r}_N \equiv \sum_{i=1}^{N} m_i\vec{r}_i. \qquad (1)$$

2b. A Typical Magnetic Dipole. The most common dipole moment vector is the magnetic dipole: two equal but opposite magnetic poles separated by a distance ℓ. For example, most of the external effects of a small bar magnet, such as a compass needle, can be viewed as resulting from equal but opposite poles at its two ends. With the strength of the

[1]See "Electric Dipoles" (MISN-0-120).
[2]Observing a magnetic monopole would be like finding only a "North" pole or only a "South" pole of a magnet. In Nature, North poles and South poles are always found together in pairs.
[3]See "Magnetic Monopole" (MISN-0-140).
[4]Observing a magnetic monopole would be like finding only a "North" pole or only a "South" pole of a magnet. In nature, North poles and South poles are always found together.

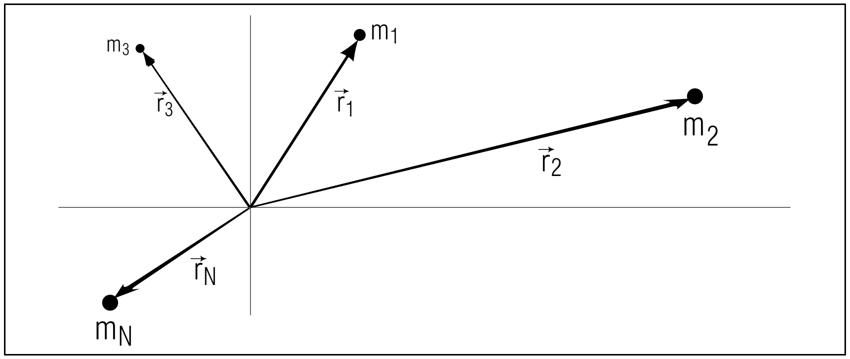


Figure 1. Magnetic monopoles m_i located at positions $\vec{r}_i$.

poles taken as $+m$ and $-m$, the dipole moment of this magnet is found from the general definition, Eq. (1). It is:

$$\vec{\mu} = m\vec{r}_1 - m\vec{r}_2 = m(\vec{r}_2 - \vec{r}_1), \qquad (2)$$

where $\vec{r}_1 - \vec{r}_2 = \vec{l}$ is the vector separation of the two poles. The magnetic dipole moment can thus be written:

$$\vec{\mu} = m\vec{\ell}, \qquad (3)$$

where by definition $\vec{l}$ points from the South pole to the North pole ($-m$ to m, see Fig. 2). Eq. (3) is analogous to the expression $\vec{p} = q\vec{l}$ for the electric dipole moment. As in that case, the magnetic dipole moment of two equal but opposite poles is independent of the coordinate system used to describe it: it is a property of the system.

3. Magnetic Field due to a Dipole

The electric dipole analogy is not strictly correct but it can provide us with the correct expression for the magnetic field produced by a magnetic dipole. By the same method used for the electric dipole,[5] the magnetic field due to a point magnetic dipole (i.e. a dipole whose size, ℓ, is negligible compared with the distance r to the point where the field of the dipole is observed) is:[6]

$$\vec{B}(\vec{r}) = k_m \left[\frac{(3\vec{\mu}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{\mu}}{r^3}\right], \qquad (\ell \ll r). \qquad (4)$$

[5]See "Electric Dipoles" (MISN-0-120).
[6]$k_m \equiv 10^{-7}\,\mathrm{N/A^2} = 10^{-7}\,\mathrm{kg\,m/C^2}$ *Help: [S-1]*

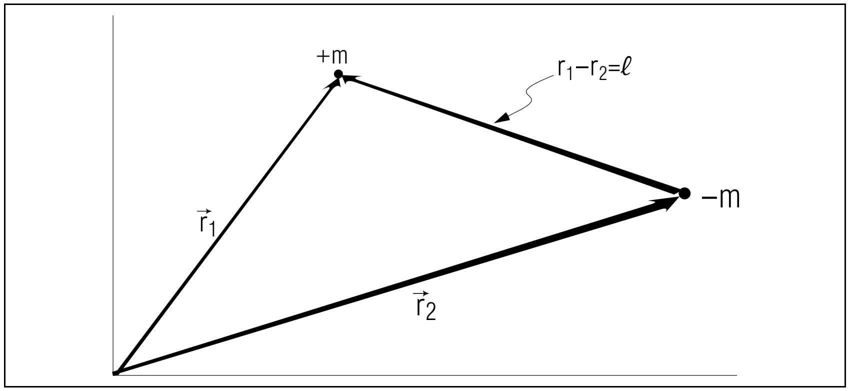


Figure 2. A common magnetic dipole with dipole moment $\vec{\mu} = m\vec{l}$.

The field lines for a dipole oriented along the z-axis are indicated in Fig. 3.

4. Existence of Magnetic Dipoles

4a. The Simplest Magnetic Structure is the Dipole. In electricity, the isolated charge q can exist by itself; but in magnetism, isolated magnetic poles have never been observed. The simplest magnetic structure is the magnetic dipole, which is characterized by the magnetic dipole moment $\vec{\mu}$. The most familiar example of a magnetic dipole is a bar magnet. Iron filings sprinkled on a sheet of paper placed over a bar magnet

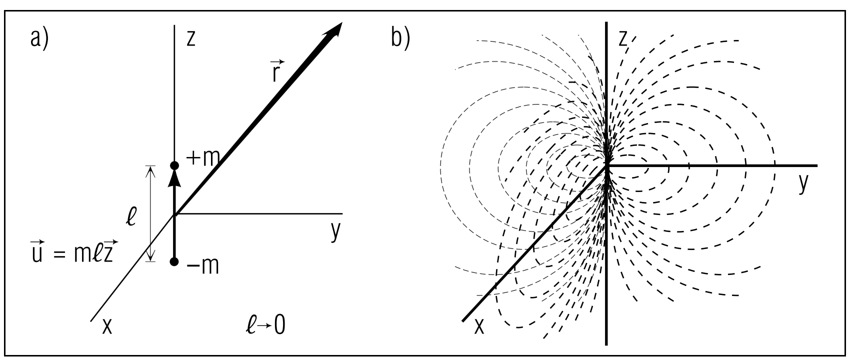


Figure 3. (a) A magnetic dipole oriented in the z-direction; (b) Magnetic field lines around this dipole.

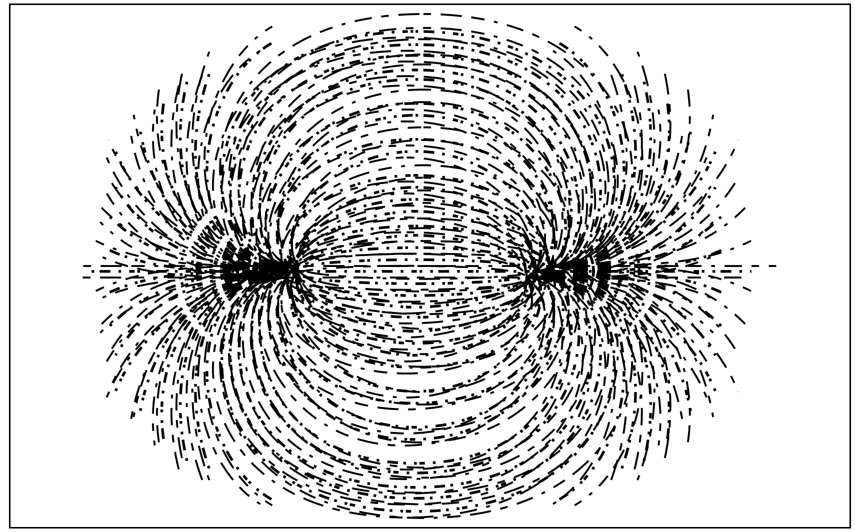


Figure 4. Iron filings placed of a sheet of paper over a bar magnet help in visualizing the magnetic field lines.

produce a pattern that suggests the dipole may be viewed as two magnetic poles, one at each end of the magnet (see Fig. 4). However, all attempts to isolate these poles fail. If the bar magnet is cut in half, the halves also turn out to be magnetic dipoles and not isolated poles.

4b. A Current Loop is a Magnetic Dipole. The explanation for the behavior of the bar magnet can be approached by first examining the magnetic field of a circular loop of current. The magnetic field produced by a circular loop of current is identical to that produced by a point

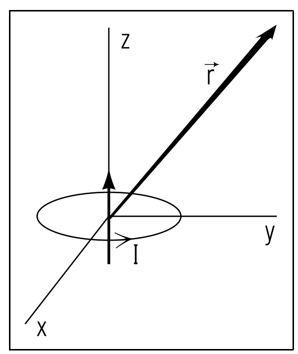


Figure 5. The dipole moment of a circular current loop is perpendicular to the plane of the loop.

magnetic dipole (see Eq. (4) and Fig. 3), so the loop is also a dipole.

Consider a current loop in the x-y plane of area A carrying a current I in the direction indicated in Fig. 5. The loop has a dipole moment along the normal ($\hat{n}$) to the plane of the loop given by $\vec{\mu} = IA\,\hat{n}$. The direction of the normal is given by a right hand rule: when the fingers of the right hand curl in the direction of the current, the thumb points along the normal.

4c. A Bar Magnet Consists of Tiny Current Loops. The fact that a current loop is a magnetic dipole suggests that the real sources of the dipole field produced by a bar magnet are tiny currents at the atomic level. These currents are produced by orbital electrons in the atoms of the bar magnet. In a bar magnet, the tiny magnetic dipole moments are aligned such that they add together to produce a strong magnetic field.[7] Now it is apparent why the magnet always has two poles: no matter how it is cut, it will always still contain the tiny current loops which give it its dipole field.

5. Dipole In External Field

5a. Torque on a Magnetic Dipole. A magnetic dipole placed in an external magnetic field experiences a torque. The torque on a magnetic dipole $\vec{\mu}$ placed in an external magnetic field $\vec{B}$ is

$$\vec{\tau} = \vec{\mu} \times \vec{B}\,. \tag{5}$$

Compare this expression to that for an electric dipole in an electric field:

$$\vec{\tau} = \vec{p} \times \vec{E}. \tag{6}$$

If $\vec{\mu}$ makes an angle θ with the external magnetic field as shown in Fig. 6, then:

$$\tau = uB\sin\theta. \tag{7}$$

Thus the torque on the current loop (or any magnetic dipole) is a maximum for $\theta = 90^\circ$ and zero for $\theta = 0^\circ$.

5b. Work Done on a Magnetic Dipole. Since a magnetic dipole placed in an external magnetic field experiences a torque, work (positive or negative) must be done by an external agent in order to change the orientation of the dipole.

[7]See "Magnetic Fields in Bulk Matter: Magnets" (MISN-0-141).

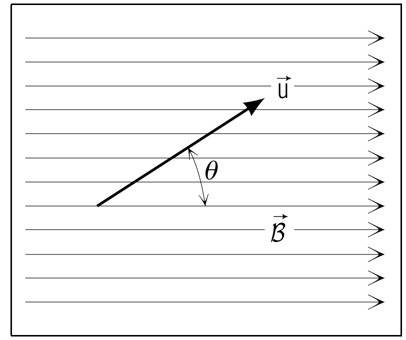


Figure 6. A dipole in an external magnetic field.

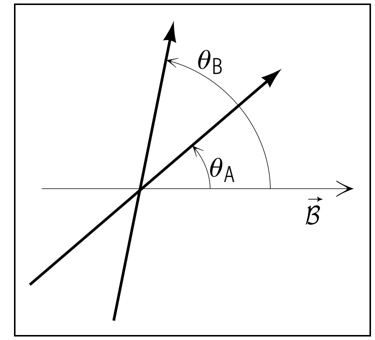


Figure 7. A dipole is rotated in a magnetic Field.

Let us calculate how much work is done by the field when rotating the dipole from angle θ_A to θ_B as shown in Fig. 7. Since the forces vary, we must integrate $\vec{F} \cdot d\vec{s}$, which can be simplified to[8]

$$\begin{aligned} W_{A,B} &= \int_{\theta_A}^{\theta_B} \tau\, d\theta \qquad (8)\\ &= \mu B \int_{\theta_A}^{\theta_B} \sin\theta\, d\theta \qquad (9)\\ &= -\mu B(\cos\theta_B - \cos\theta_A). \qquad (10)\end{aligned}$$

This can be written as:[9]

$$W_{A,B} = (-\vec{\mu}\cdot\vec{B})|_{\theta=\theta_B} - (-\mu\cdot\vec{B})|_{\theta=\theta_A}. \tag{11}$$

5c. Potential Energy of a Magnetic Dipole. A magnetic dipole has potential energy since the work depends only upon the initial and final positions, not the path taken. As was done for the electric dipole, we will set the potential energy Ep equal to zero when the dipole is at right angles to the field, i.e., when $\theta = 90^\circ$. Thus the magnetic potential energy of a magnetic dipole is

$$E_p(\theta) = -W_{90^\circ,\theta} = uB(\cos 90^\circ - \cos\theta).$$

or:

$$E_p = -\mu B\cos\theta = -\vec{\mu}\cdot\vec{B}. \tag{12}$$

[8]See "Electric Dipoles" (MISN-0-120) for the steps in this simplification.
[9]This shows that the units of $\vec{\mu}$ are J/T [S-2].

6. Electric/Magnetic Dipole Analogy

The table below summarizes some relationships that are mathematically similar for electric and magnetic dipoles. In addition, the dimensions of the quantities (in terms of the fundamental dimensions M, L, T, Q) are given as reminders of both the similarities and the differences.

Quantity/Property	**Electric**	**Magnetic**
Monopole Charge	q [Q]	m $[\mathrm{LT^{-1}Q^{-1}}]$
Dipole Moment	$\vec{p} = q\vec{\ell}$ $[QL]$	$\vec{\mu} = m\vec{\ell}$ $[\mathrm{L^2T^{-1}Q^{-1}}]$
Field at $\vec{r}$ due to point dipole at origin	$\vec{E} = k_e\left[\frac{(3\vec{p}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{p}}{r^3}\right]$ $[\mathrm{MLT^{-2}Q^{-1}}]$	$\vec{B} = k_m\left[\frac{(3\vec{\mu}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{\mu}}{r^3}\right]$ $[\mathrm{MT^{-1}Q}]$
Force on monopole at $\vec{r}$ due to dipole at origin	$\vec{F} = q\vec{E}$, using above $\vec{E}$ $[\mathrm{MLT^{-2}}]$	$\vec{F} = m\vec{B}$, using above $\vec{B}$ $[\mathrm{MLT^{-2}}]$
Torque exerted on dipole placed in external field	$\vec{\tau} = \vec{p}\times\vec{E}_{\mathrm{ext}}$ $[\mathrm{ML^2T^{-2}}]$	$\vec{\tau} = \vec{\mu}\times\vec{B}_{\mathrm{ext}}$ $[\mathrm{ML^2T^{-2}}]$
Potential Energy of dipole in external field	$E_p = -\vec{p}\cdot\vec{E}_{\mathrm{ext}}$ $[\mathrm{ML^2T^{-2}}]$	$E_p = -\vec{\mu}\cdot\vec{B}_{\mathrm{ext}}$ $[\mathrm{ML^2T^{-2}}]$

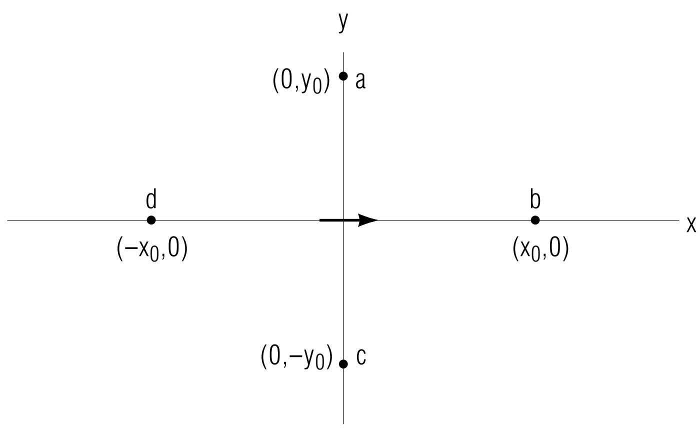
Acknowledgments

This module was based on an earlier module by P. Sojka, J. Kovacs, and P. Signell. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

PROBLEM SUPPLEMENT

Note: Problems 9 and 10 also occur in this module's *Model Exam*.

1. A point dipole, with magnetic moment $\vec{\mu}$, is in an external magnetic filed $B_0\hat{y}$ where B_0 is a constant.
   a. Determine the torque on the dipole for each of these conditions:
      (i) $\vec{\mu} = \mu\hat{x}$.
      (ii) $\vec{\mu} = \mu\hat{y}$.
      (iii) $\vec{\mu} = -\mu\hat{x}$.
      (iv) $\vec{\mu} = -\mu\hat{y}$.
   b. Determine the potential energy of the dipole in each of the four orientations given in part (a).
   c. Determine the work done in turning the dipole:
      (i) from $\hat{x}$ to $\hat{y}$.
      (ii) from $\hat{x}$ to $-\hat{x}$.
      (iii) from $\hat{y}$ to $-\hat{x}$.
2. Consider a point dipole, at the origin, with magnetic moment $\vec{\mu} = (-10^{-2}\,\mathrm{m^2s^{-1}C})\,\hat{x}$ in a uniform external magnetic field $\vec{B} = 50\,\mathrm{T}\,\hat{y}$. Determine:
   a. the potential energy of the dipole.
   b. the torque on the dipole.
3. A magnetic dipole $\vec{\mu} = \mu\hat{x}$ is located at the origin of a coordinate system. Determine the magnetic field(magnitude and direction) at each of the points a, b, c, d shown in the sketch:



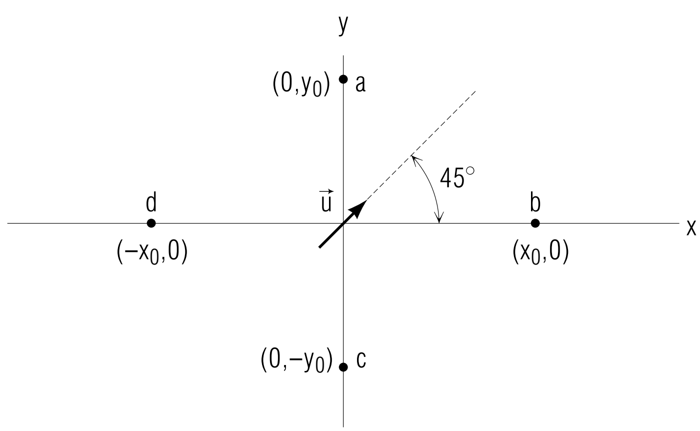
4. Determine the magnetic field at each of the four points in Problem (3) if $\vec{\mu} = 2\,(\mathrm{J\,T^{-1}})\,\hat{x}$ and $x_0 = 0.01\,\mathrm{m}$, $y_0 = 0.01\,\mathrm{m}$.

5. The magnetic moment $\vec{\mu}$ of a point dipole in a constant magnetic field $\vec{B}_0$ makes an angle of 30° with the field.

   a. Determine the potential energy of the dipole.

   b. Determine the torque on the dipole.

6. Determine the magnetic field at points a, b, c, d shown below if the magnetic dipole makes an angle of 45° with the positive x and y axes in the x-y plane.

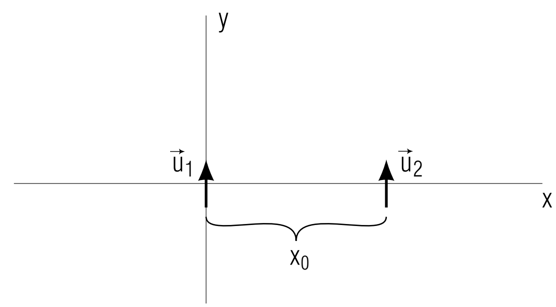


7. A point dipole with magnetic moment $\vec{\mu}$ is in an external field $\vec{B} = B_0\hat{x}$. Since torque and potential energy have the same units, determine the direction(s) in which the dipole must be pointing if its torque and potential energy are to be "equal." Determine the work that must now be done to reorient the dipole so it points in the direction of the field.

8. A dipole $\vec{\mu}_1 = \mu_1\hat{y}$ is located at the origin, while another, $\vec{\mu}_2 = \mu_2\hat{y}$, is located at the point $x = x_0$, $y = 0$.

   a. Write down the expression for the magnetic field at the site of $\vec{\mu}_2$ due to $\vec{\mu}_1$.

   b. Treating the field due to $\vec{\mu}_1$ as a field external to $\vec{\mu}_2$ (as it is), find the potential energy of $\vec{\mu}_2$ in this field, hence finding the *interaction potential energy* of these two dipoles.



   c. Find the interaction potential energy if $x_0 = 0.05\,\mathrm{m}$, $\mu_1 = 3\,\mathrm{J\,T^{-1}}$ and $\mu_2 = 2\,\mathrm{J\,T^{-1}}$. Put in the proper units of all quantities and assure yourself that all of the units combine properly to give the appropriate units for your answer.

9. A point dipole with magnetic moment $\vec{\mu} = \mu_x\hat{x} + \mu_y\hat{y}$ is in an external magnetic field $\vec{B} = B_0\hat{x}$.

   a. What is the torque on the dipole?

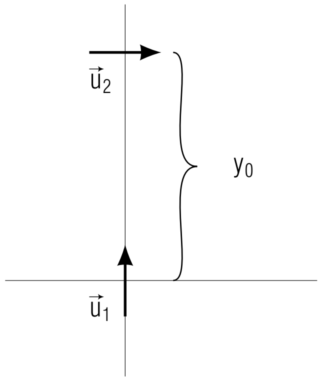
   b. Find the potential energy of the dipole.

   c. How much work must be done to reorient the dipole so it points in the direction of the field?

10. A point dipole $\vec{\mu}_1 = \mu_1\hat{y}$ is at the origin, while at $x = 0$, $y = y_0$ is located another dipole $\vec{\mu}_2 = \mu_2\hat{x}$.

a. What is the magnetic field at the site of $\vec{\mu}_2$ due to $\vec{\mu}_1$?

b. Find the energy of the system using $y_0 = .04\,\mathrm{m}$, $\mu_1 = .5\,\mathrm{J\,T^{-1}}$ and $\mu_2 = .6\,\mathrm{J\,T^{-1}}$.



Brief Answers:

1. a. $\vec{\tau} = \vec{\mu} \times \vec{B}$

   (i) $\vec{\tau} = \mu\hat{x} \times B_0\hat{y} = \mu B_0\hat{z}$.

   (ii) $\vec{\tau} = \mu\hat{y} \times B_0\hat{y} = 0$.

   (iii) $\vec{\tau} = -\mu\hat{x} \times B_0\hat{y} = -\mu B_0\hat{z}$.

   (iv) $\vec{\tau} = -\mu\hat{y} \times B_0\hat{y} = 0$.

   b. $E_p = -\vec{\mu} \cdot \vec{B}$.

   (i) $E_p = -\mu B_0\hat{x} \cdot \hat{y} = 0$.

   (ii) $E_p = -\mu B_0\hat{y} \cdot \hat{y} = -\mu B_0$.

   (iii) $E_p = \mu B_0\hat{x} \cdot \hat{y} = 0$.

   (iv) $E_p = \mu B_0\hat{y} \cdot \hat{y} = \mu B_0$.

   c. $W = (-\vec{\mu} \cdot \vec{B})_{\theta=\theta_B} - (-\vec{\mu} \cdot \vec{B})_{\theta=\theta_A}$.

   (i) $W = -\mu B_0 - 0 = -\mu B_0$.

   (ii) $W = 0 - 0 = 0$.

   (iii) $W = 0 - (-\mu B_0) = \mu B_0$.

2. a. $E_p = -\vec{\mu} \cdot \vec{B} = -(-10^{-2}\,\mathrm{m^2\,s^{-1}\,C})\,\hat{x} \cdot (50\,\mathrm{T})\;\hat{y} = 0$.

   b. $\vec{\tau} = \vec{\mu} \times \vec{B} = (-10^{-2}\,\mathrm{m^2\,s^{-1}\,C})\,\hat{x} \times (50\,\mathrm{T})\hat{y} = -0.5\,\mathrm{N\,m}\,\hat{z}$, where we have used: $\mathrm{T} = \mathrm{kg\,s^{-1}\,C^{-1}}$.

3. $\vec{B} = k_m \left[\dfrac{3\mu\hat{x} \cdot \hat{r}}{r^5} - \dfrac{\mu\hat{x}}{r^3}\right]$.

   At a: $\vec{r} = y_0\hat{y}$; $\vec{B} = k_m \left[\dfrac{3\mu y_0(\hat{x} \cdot \hat{y})y_0\hat{y}}{y_0^5} - \dfrac{\mu\hat{x}}{y_0^3}\right] = -k_m \dfrac{\mu}{y_0^3}\hat{x}$.

   At b: $\vec{r} = x_0\hat{x}$; $\vec{B} = k_m \left[\dfrac{3\mu x_0(\hat{x} \cdot \hat{x})x_0\hat{x}}{x_0^5} - \dfrac{\mu\hat{x}}{x_0^3}\right] = 2k_m \dfrac{\mu}{x_0^3}\hat{x}$.

   At c: $\vec{r} = -y_0\hat{y}$; $\vec{B} = k_m \left[\dfrac{-3\mu y_0(\hat{x} \cdot \hat{y})(-y_0\hat{y})}{y_0^5} - \dfrac{-\mu\hat{x}}{y_0^3}\right] = -k_m \dfrac{\mu}{y_0^3}\hat{x}$.

   At d: $\vec{r} = -x_0\hat{x}$; $\vec{B} = k_m \left[\dfrac{-3\mu x_0(\hat{x} \cdot \hat{x})(-x_0\hat{x})}{x_0^5} - \dfrac{\mu\hat{x}}{x_0^3}\right] = k_m \dfrac{\mu}{x_0^3}\hat{x}$.

4. At a:
   $\vec{B} = -k_m \dfrac{\mu\hat{x}}{y_0^3} = -\dfrac{(10^{-7}\,\mathrm{T\,m\,C^{-1}s})(2\,\mathrm{m^2s^{-1}C})\,\hat{x}}{(0.01\,\mathrm{m})^3} = -0.2\,\hat{x}\,\mathrm{T}$.

   At b: $\vec{B} = k_m \dfrac{\mu\hat{x}}{x_0^3} = \dfrac{(10^{-7}\,\mathrm{T\,m\,C^{-1}s})(2\,\mathrm{m^2s^{-1}C})\,\hat{x}}{(1/2)(0.01\,\mathrm{m})^3} = 0.4\hat{x}\,\mathrm{T}$.

   At c: $\vec{B} = -0.2\,\hat{x}\,\mathrm{T}$.

   At d: $\vec{B} = +0.4\hat{x}\,\mathrm{T}$.

5. a. $E_p = -\vec{\mu} \cdot \vec{B} = -\mu B_0 \cos 30^\circ = -0.866\mu B_0$.

   b. $|\vec{\tau}| = |\vec{\mu} \times \vec{B}| = \mu B_0 \sin 30^\circ = 0.5\mu B_0$.

6. $\vec{\mu} = \mu \dfrac{\hat{x} + \hat{y}}{\sqrt{2}}$

   At a:
   $\vec{B} = k_m \left[\dfrac{3\mu y_0\hat{y}}{\sqrt{2}y_0^5} - \dfrac{\mu}{y_0^3}\dfrac{\hat{x} + \hat{y}}{\sqrt{2}}\right] = k_m \dfrac{\sqrt{2}\mu}{2y_0^3}(-\hat{x} + 2\hat{y})$

7. $\tau = \mu B_0 \sin\theta$; $E_p = -\mu B_0 \cos\theta$

   so $\sin\theta = -\cos\theta$ which means $\theta = 135^\circ$ or 315°

   $W = -\mu B_0(\cos 0^\circ - \cos 135^\circ) = -1.707\mu B_0$.

   $W = -\mu B_0(\cos 0^\circ - \cos 315^\circ) = -0.239\mu B_0$.

8. a. $\vec{\mu} = \mu\hat{y}; \qquad \vec{r} = x_0\hat{x}$

$$\vec{B} = k_m \left[\frac{(3\vec{\mu}\cdot\vec{r})\vec{r}}{r^5} - \frac{\mu}{r^5} \right] = k_m \left[0 - \frac{\mu\hat{y}}{x_0^3} \right] = k_m \frac{\mu}{x_0^3}\,\hat{y}.$$

b. $E_p = -\vec{\mu}\cdot\vec{B} = -\mu_2\hat{y}\cdot\left[-k_m \frac{\mu_1}{x_0^3}\,\hat{y} \right] = k_m \frac{\mu_1\mu_2}{x_0^3}.$

c. $E_p = \dfrac{(10^{-7}\,\mathrm{m\,kg\,C^{-2}})(2\,\mathrm{m^2 s^{-1} C})(3\,\mathrm{m^2 s^{-1} C})}{(5\times 10^{-2}\,\mathrm{m})^3} = 4.8\times 10^{-3}\,\mathrm{J}.$

9. a. $\vec{\tau} = \mu_y B_0 \hat{z}.$

b. $E_p = -\mu_x B_0.$

c. $W = -\mu B_0 + \mu_x B_0,$

$\mu = \sqrt{\mu_x^2 + \mu_y^2}.$

10. a. $\vec{B} = 2k_m \dfrac{\mu_1}{y_0^3}\,\hat{y}.$

b. Zero.

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from TX-3)*

$\mathrm{T} = \text{tesla} = \dfrac{\mathrm{m\,kg\,C^{-2}}}{\mathrm{m\,C^{-1}\,s}} = \mathrm{kg\,s^{-1}\,C^{-1}}.$

T is the SI unit for magnetic fields.

S-2 *(from TX-5b)*

$W = -\mu B(\cos\theta_B - \cos\theta_A)$

J = (units of μ) T

units of $\mu = \mathrm{J\,T^{-1}} = (\mathrm{kg\,m^2\,s^{-2}})/(\mathrm{kg\,s^2\,C^{-1}}) = \mathrm{m^2\,s^{-1}\,C}.$

MODEL EXAM

1. Explain how there can be magnetic dipoles in nature while there don't seem to be magnetic monopoles.

2. For both electric and magnetic dipoles, write the corresponding expressions for torque, work, potential energy and field.

3. A point dipole with magnetic moment $\vec{\mu} = \mu_x \hat{x} + \mu_y \hat{y}$ is in an external magnetic field $\vec{B} = B_0 \hat{x}$.

   a. What is the torque on the dipole?

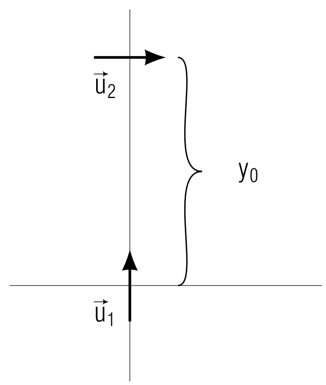
   b. Find the potential energy of the dipole.

   c. How much work must be done to reorient the dipole so it points in the direction of the field?

4. A point dipole $\vec{\mu}_1 = \mu_1 \hat{y}$ is at the origin, while at $x = 0$, $y = y_0$ is located another dipole $\vec{\mu}_2 = \mu_2 \hat{x}$.

   a. What is the magnetic field at the site of $\vec{\mu}_2$ due to $\vec{\mu}_1$?

   b. Find the energy of the system using $y_0 = 0.04\,\text{m}$, $\mu_1 = 0.5\,\text{J}\,\text{T}^{-1}$ and $\mu_2 = 0.6\,\text{J}\,\text{T}^{-1}$.



Brief Answers:

1. See this module's *text*.

2. See this module's *text*.

3. See Problem 9 in this module's *Problem Supplement*.

4. See Problem 10 in this module's *Problem Supplement*.