PHYSNET

MISN-0-493

NEWTONIAN MECHANICS - SINGLE PARTICLE

Classical Mechanics

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

NEWTONIAN MECHANICS - SINGLE PARTICLE

by

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Input Skills:

1. Manipulate vectors analytically (MISN-0-492).
2. Evaluate the gradient of a given function (MISN-0-492).
3. Evaluate simple line integrals (MISN-0-492).

Output Skills (Knowledge):

K1. Briefly outline the conceptual foundations of Newtonian mechanics including statements about: absolute space, absolute time, Galilean relativity and inertial frames, particles, inertial mass and force, Newton's three laws of particle motion.

K2. Derive these laws for a single particle starting from Newton's second law: conservation of linear momentum, conservation of angular momentum, conservation of kinetic energy, conservation of mechanical energy.

K3. State the relations between: impulse and momentum, torque and angular momentum, angular impulse and angular momentum, power and kinetic energy, work and kinetic energy, non-conservative power and mechanical energy, non-conservative work and mechanical energy.

Output Skills (Problem Solving):

S1. Solve problems of the types assigned in the module's *Procedures* section.

External Resources (Required):

1. J. Marion, *Classical Dynamics*, Academic Press (1988).
2. D. T. Greenwood, *Principles of Dynamics*, Prentice-Hall (1965).
3. R. T. Weidener and Sells, *Elementary Classical Physics*, Vol. I, Allyn and Bacon (1965).

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1. Introduction

This unit reviews the fundamentals of Newtonian mechanics as embodied in Newton's famous three laws of particle motion. These three laws were stated by Newton in his *Principia* of 1687 in a form similar to that commonly used today. There is hardly a physicist now alive who would dispute the importance of these three laws and the attendant concepts enunciated by Newton. Despite this there is even today considerable controversy as to the significance, content and meaning (or lack of) of each of the three laws. In fact a quick look at a half dozen mechanics texts will reveal a half dozen approaches to Newton's laws.

The central issue in the controversy is the status of the second law. Newton, in his principia, failed to make clear the meaning of force. Consequently, many authors seize upon the second law as a definition of force while others claim that it is more than a mere definition. Its current usage as a tool for predicting the future behavior of a particle from a knowledge of the particles mechanical environment would certainly indicate that the second law is more than a definition. It implies that it is a fundamental law of nature which gives the time evolution of a mechanical system. This latter point of view will be adopted in this course.

Also involved in the controversy is the cause-and-effect relationship between force and acceleration. Does the force cause the acceleration or vice versa. Treating Newton's second law as a definition of force implies that without an acceleration there is no force, i.e. acceleration is the cause and force the effect. However, it is common practice to say that one exerts a force on a building, for example, when he pushes against it even though there is no acceleration. In such a case the effect is present even though there is no cause. To avoid such situations, it seems preferable to adopt the contrary point of view - that force is the cause and acceleration the effect. One is then in a position to consider several forces acting simultaneously on an object and saying that the total force (vector sum) is the cause of the acceleration of the object.

This unit will attempt to provide you with one (of several possible) reasonably good conceptual foundation in Newtonian mechanics utilizing

the time evolution (predictor) role of the second law. Some fundamental and useful consequences of the second law will also be covered along with some applications to single particle motion.

2. Procedures

1. Read Symon, section 1-4, for a good overview of the controversy that continues to surround the conceptual foundations of Newtonian mechanics.

   Optional reading (giving varying textbook viewpoints): Konopinski, pp. 30 - 40 (elaborate discussion)

   Goldstein, p. 1 (dispenses with the whole problem in a few sentences)

   Marion, pp. 43 - 54 (adopts a point of view in which the author of this study guide does not concur).

   Arthur and Fenster, p. 94 (makes a few statements to which the author of this study guide does concur.)

   Greenwood, pp. 19 - 25 (an engineering approach which has some very definite advantages).

   a. Newton assumed that there was a rigid underlying framework - a sort of stage - in which particle motion occurred. This so-called absolute space was unchanging and unchangeable. It was relative to this framework that the motion of particles was to be described by the three laws of particle motion. Newton also assumed an absolute, unchanging, uniformly flowing time.

      Although Newton assumed the existence of absolute space, he realized that he could not identify it by mechanical means only. It was well known that uniform relative motion did not alter the observed behavior of mechanical systems. In fact, Newton's three laws (to be discussed later) have the same form in all reference frames that move with constant velocity (and are non-rotating) relative to absolute space. Such reference frames are known as inertial frames and the invariance of the laws of mechanics under transformations from one inertial frame to another is known as Galilean (or Newtonian) relativity.

      Since the laws of mechanics as enunciated by Newton are the same in all inertial frames, it is impossible by mechanical means alone to identify that one inertial frame that is absolute space. Newton and his contemporaries felt that some other physical phenomenon -

perhaps light or some aspect of electricity and magnetism - would be capable of pinpointing absolute space. This attitude prevailed up through the early 1900's. In fact, the unsuccessful attempts in the late 1800's to determine the velocity of the earth relative to the "aether" were in effect attempts to pinpoint absolute space. The failure to do so ultimately lead to the abandonment of the concept of absolute space and the development of the theory of special relativity by Einstein. Despite our present day recognition of the non-existence of absolute space, it is still convenient to use the concept in classical mechanical discussions.

b. *Particle* - the simplest definition for a particle in classical mechanics is any object whose size and internal structure are unimportant for the problem at hand. Thus, the earth is a particle when discussing its motion around the sun but an atom is not a particle when discussing the mechanism whereby an atom emits radiation.

Mass - There are two good alternative ways of quantifying the concept of mass. One is a more or less standard discussion first presented by Mach and depends on the qualitative implications of Newton's third law. The other method utilizes a device known as an inertial balance and seems to stand independent of Newton's laws. We will use the latter procedure because of its independence of Newton's laws.

Read Weidner and Sells, pp. 118 - 121. (Inertial balance)

Optional reading: Arthur and Fenster, pp. 86 - 87 (good presentation of Mach's argument)

It should be noted that the inertial balance compares masses. By subdividing or combining masses, one can in principle compare any mass to a standard mass. It should also be noted that mass is a scalar quantity.

Exercise: Imagine having available a good inertial balance and a large stockpile of brass (along with the necessary tools to cut and machine it). Determine in your own mind the proper way to go about constructing a set of masses (as used in a chemical balance).

Force - Once one has at his disposal the concepts of an inertial frame, it is easy to qualitatively define force: it is any agency that tends to cause a particle to accelerate as viewed from an inertial frame. To quantify that definition, one needs also the concept of mass. Then the quantitative definition of force is as follows: If a (single) force (or agency) is allowed to act on the standard mass, then the value of the force is equal to the product of the resulting

acceleration and the mass of the standard. It should be noted that this definition makes no claim about: 1. the quantitative effect of the force on an arbitrary mass, and 2. the quantitative effect of several forces on a mass. When defining force, it is preferable to give both the qualitative and quantitative statements.

c. Read Marion, first four paragraphs of Section 2.2 only, for conventional statements of Newton's three laws.

Optional: For statements of Newton's laws which are more nearly in Newton's own words, see pages 111, 113 and 114 of *Principles of Relativity Physics* by J. L. Anderson. (Newton, of course, wrote the Principia in latin so these are English translations.)

Comments on the first law: Newton's first law along with the qualitative definition of force implies that it is both necessary and sufficient that a force act on a particle in order for the particle to accelerate relative to an inertial frame. Consequently, it is quite common to use the first law as a test to identify an inertial frame. That is, if an arbitrary, free (isolated) particle is observed to travel at constant speed in a straight line, the observer(s) is at rest relative to an inertial frame.

Comments on the second law: The manner in which this law is stated (both in Marion and in Anderson, as well as other texts) connotes a predictor role rather than a definer role for the second law. If Newton had intended for it to be a definition he would have simply said "Force is . . . " or something similar. In fact, Newton's second law goes far beyond a definition. It does the two things that our force definition did not do:

1. It determines the effect of a (single) force on an arbitrary mass.
2. It determines the effect of the simultaneous action of several forces upon a single particle.

Concerning this second point, if several forces act simultaneously on a particle, then according to Newton's second law each force induces in the particle an acceleration. Since acceleration is a vector quantity, the total acceleration of the particle is the vector sum of the separate accelerations. Thus Newton's law is, in effect, saying that when several forces act on a particle the forces can be added as vectors. The total force thus obtained determines the total acceleration.

▷ Exercise - For each of the items in Output Skill K1, organize in your mind what you would write about it if asked to do so within the stated context. Your comments should be a condensation of what has been presented so far in the procedures.

Comment: It should be noted that momentum is defined for convenience to be the product of mass times velocity.

2. There are four sets of laws that follow as theorems from Newton's second law. They are outlined below. The four sets have a great deal of similarity which you should try to discern. For additional details see Marion, section 2.5, and your intermediate mechanics text.

   a. Newton's second law:
$$\vec{F} = \frac{d\vec{p}}{dt}\,.$$
   Integrate on time:
$$\int_{t_1}^{t_2} \vec{F} dt = \vec{p}_2 - \vec{p}_1 = \Delta\vec{p}.$$
   Definition:
$$\text{Impulse} \equiv \vec{I} \equiv \int_{t_1}^{t_2} \vec{F} dt\,.$$
   Result: Impulse during specified time interval equals change in momentum during that time
$$\vec{I} = \Delta\vec{p}\,.$$
   Conservation of linear momentum: (From above result it is clear that) the momentum of a particle is conserved (constant, unchanged) if the impulse is zero.

   Special case: Obviously $\vec{F} = 0 \Rightarrow \vec{I} = 0 \Rightarrow \vec{p}_2 = \vec{p}_1$.

   b. Cross $\vec{r}$ (particle position) into Newton's second law:
$$\vec{r} \times \vec{F} = \vec{r} \times \frac{d\vec{p}}{dt}\,.$$
   Definition:
$$\text{Torque} \equiv \vec{N} \equiv \vec{r} \times \vec{F}\,.$$
   Observe:
$$\frac{d}{dt}(\vec{r} \times \vec{p}) = \vec{r} \times \frac{d\vec{p}}{dt} + \vec{v} \times (m\vec{v}) = \vec{r} \times \frac{d\vec{p}}{dt}\,.$$

   Definition:
$$\text{Angular momentum} \equiv \vec{L} \equiv \vec{r} \times \vec{p}\,.$$
   Result: Torque equals time rate of change of angular momentum:
$$\vec{N} = \frac{d\vec{L}}{dt}\,.$$
   Note: compare remainder of part (b) with part (a).

   Integrate on time:
$$\int_{t_1}^{t_2} \vec{N} dt = \vec{L}_2 - \vec{L}_1 = \Delta\vec{L}\,.$$
   Definition:
$$\text{Angular impulse} \equiv \vec{G} \equiv \int_{t_1}^{t_2} \vec{N} dt\,.$$
   Result: Angular impulse during specified time interval equals change in angular momentum during that time interval:
$$\vec{G} = \Delta\vec{L}\,.$$
   Conservation of angular momentum: the angular momentum of a particle is conserved if the angular impulse is zero.

   Special case: $\vec{N} = 0 \Rightarrow \vec{G} = 0 \Rightarrow \vec{L}_2 = \vec{L}_1$.

   c. Dot $\vec{v}$ (particle velocity) into Newton's second law:
$$\vec{v} \cdot \vec{F} = \vec{v} \cdot \frac{d\vec{p}}{dt}\,.$$
   Observe:
$$\vec{F} \cdot \vec{v} \approx \vec{F} \cdot \left(\frac{\Delta\vec{r}}{\Delta t}\right) = \frac{\vec{F} \cdot \Delta\vec{r}}{\Delta t}\,.$$
   Definition: Work done on particle during time Δt is $\delta W = \vec{F} \cdot \Delta\vec{r}$ where $\Delta\vec{r}$ is the displacement of the particle during Δt.

   Definition: Power = time rate at which work is done = $P = \delta W/\Delta t$.

   Observe:
$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \frac{d}{dt}\left(\frac{1}{2}m\vec{v} \cdot \vec{v}\right)$$
$$= \frac{1}{2}m\left(\vec{v} \cdot \frac{d\vec{v}}{dt} + \frac{d\vec{v}}{dt} \cdot \vec{v}\right) = \vec{v} \cdot \frac{d\vec{p}}{dt}\,.$$

Definition:

$$\text{Kinetic energy} \equiv T = \frac{1}{2}mv^2 \,.$$

Result: Power equals time rate of change of kinetic energy:

$$P \equiv \frac{dT}{dt} \,.$$

Note: compare remainder of part (c) with part (a).

Integrate on time:

$$\int_{t_1}^{t_2} P dt = T_2 - T_1 = \Delta T \,.$$

Consequence of Work definition:

$$\int_{t_1}^{t_2} P dt = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot \vec{r} = W \,,$$

where W = work done on a particle from the initial position $\vec{r}_1$, to the final position $\vec{r}_2$.

Note: W involves a line integral along the path of motion of the particle.

Result: Work done on the particle during a specified time interval equals the change in kinetic energy of the particle:

$$W = \Delta T \,.$$

Conservation of kinetic energy (not extremely important but included for completeness): The kinetic energy of a particle is conserved if the work done on it is zero.

d. It is convenient to divide forces into two categories characterized as follows:

conservative—those for which the work done is independent of the path followed by the particle.

non-conservative—all others including friction and push- pull type forces.

Since for a conservative force ($\vec{F}_c$) the work done is independent of path, the work must depend only on the endpoints of the path so that:

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_c \cdot d\vec{r} = -[U(\vec{r}_2) - U(\vec{r}_1)] \,.$$

The minus sign in front is for later convenience. It can be shown that this is equivalent to:

$$\vec{F}_c = -\vec{\nabla} U \,.$$

Definition: The potential energy function associated with a conservative force $\vec{F}_c$ is that function for which:

$$\vec{F}_c = -\vec{\nabla} U \,,$$

and

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_c \cdot d\vec{r} = -[U(\vec{r}_2) - U(\vec{r}_1)] = -\Delta U \,.$$

The total force acting on a particle can thus be divided into a conservative part $\vec{F}_c$ and a non-conservative part $\vec{F}_n$:

$$\vec{F} = \vec{F}_c + \vec{F}_n \,.$$

Similar expressions exist for the power and work done by these forces:

$$P = P_c + P_n, \quad W = W_c + W_n \,.$$

Since the work done by the conservative forces is equal to the negative of the change in potential energy, these relations can be written:

$$P = -\frac{dU}{dt} + P_n, \quad W = -\Delta U + W_n \,.$$

When these are combined with the results of part c it follows that:

$$P_n = \frac{d}{dt}(T + U), \quad W_n = \Delta(T + U) \,.$$

Definition:

$$\text{Mechanical energy } \equiv E = T + U \,.$$

Result: Power expended by non-conservative forces equals the time rate of change of the mechanical energy:

$$P_n = \frac{dE}{dt} \,.$$

Result: Work done by non-conservative forces equals the change in the mechanical energy:

$$W_n = \Delta E = E_2 - E_1 \,.$$

Conservation of mechanical energy: The mechanical energy is conserved if the work done by non-conservative forces is zero.

Special case: $\vec{F}_n = 0 \Rightarrow W_n = 0 \Rightarrow E_2 = E_1$.

Final comment: The preceding is intended to be an outline only. You should not attempt to memorize but instead use it as a guide to help you see how to derive the laws listed in Output Skills K2 and K3.

3. Read Section 2.4 up to Example 2.8.

   Work problems 2-2, 2-3, 2-12, 2-15 in Marion.

   Note: For some one-dimensional problems, where it is convenient to use the expression:

$$q = v\frac{dv}{dx}\,,$$

   which is easily verified since $v = dx/dt$.

   ▷ Work problems 3-1, 3-2, 3-14 in Greenwood.

Acknowledgments

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