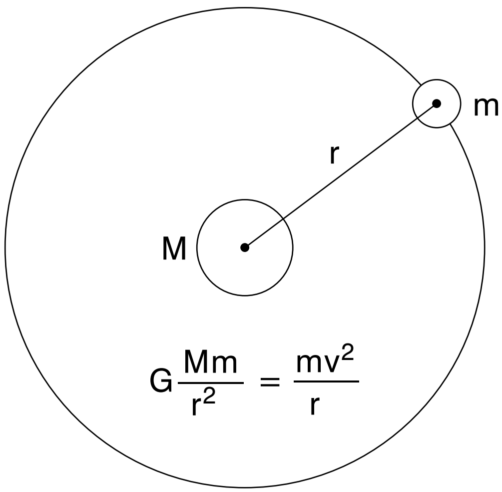
PHYSNET

MISN-0-410

# THE GRAVITATIONAL FORCE




Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI



**THE GRAVITATIONAL FORCE**

by

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Title: **The Gravitational Force**

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**Input Skills**:

1. State the direction and magnitude of the acceleration of a particle moving in a circle with constant speed (MISN-0-376).
2. State four properties of the gravitational force (MISN-0-409).

**Output Skills (Knowledge)**:

K1. State the gravitational force law.

K2. Describe the gravitational force on a particle due to a spherically symmetric object.

K3. Derive in a systematic manner the relationship between $g$ and $G$.

K4. Describe a method for determining the speed of a planet from the gravitational force law.

**Output Skills (Problem Solving)**:

S1. Given a relation which involves only multiplication and division, such as the gravitational force law, use a comparison of the values of one quantity to compare the values of other corresponding quantities.

S2. For a particle acted on only by the gravitational force due to an astronomical body of given mass, relate the acceleration of the particle (or its speed along a circular orbit) to its distance from the body.

# MISN-0-410

## THE GRAVITATIONAL FORCE




**Abstract:**
Up to now we have described various forces from an entirely empirical point of view. To gain a more unified understanding of such forces and to achieve greater predictive power, we shall now examine two of the four fundamental forces which are ultimately responsible for all other forces. Thus we shall use the present unit to discuss the *gravitational* force which accounts for the interaction between all astronomical bodies, the motion of the planets and the moon, the trajectories of space vehicles, the occurrence of the tides, and the weights of objects. In the next unit we shall then discuss the *electric* force which is responsible for the interaction between all atoms and molecules.






SECT.

## A GRAVITATIONAL FORCE LAW

As discussed in units 406 and 407, the earth interacts with any object near its surface by an interaction which is called "gravitational." This interaction accounts also for the motion of artificial satellites near the earth and for the motion of the moon around the earth. It is then plausible to assume that this interaction is also responsible for the motion of the earth and of the other planets around the *sun*. In short, we are led to make the general assumption that *any* two particles in the universe interact by this gravitational interaction.

What then is the "force law" which describes this gravitational interaction? In other words, how does the gravitational force on one particle due to another depend on the properties of these particles and on the distance between them? As usual, the answer to this question is suggested by a few observations and can then be verified by investigating whether the assumed force law predicts correctly a wide range of observed phenomena.

Consider any two particles 1 and 2 interacting by gravitational interaction. (For example, as illustrated in Fig. A-1, these particles might be the moon and the earth.) As discussed in text section E of Unit 408, the force on one particle due to the other must have a direction along the line joining the particles, i.e., it must be either repulsive or attractive. But we know that the gravitational force on an object near the earth is directed toward the center of the earth and that the gravitational force on any planet is directed toward the sun. These observations suggest that the gravitational force on any particle is always directed *toward* the other particle with which it interacts. Thus we are led to make this general assumption:

$$\boxed{\text{The gravitational force is always attractive.}} \tag{A-1}$$

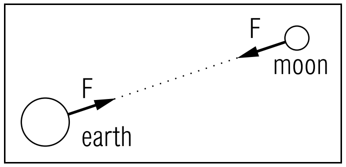



Fig. A-1: Gravitational interaction between two particles.

Since the mutual forces on the particles are related so that $\vec{F}_{1,2} = -\vec{F}_{2,1}$, the *magnitude* of the gravitational force on either particle due to the other has the same value $F = |\vec{F}_{1,2}| = |\vec{F}_{2,1}|$. We know that the gravitational force on a particle due to the earth is proportional to the mass $m$ of the particle. Hence it is plausible to assume that, for *any* two interacting particles 1 and 2, the gravitational force on either particle is proportional to its mass. Thus the magnitude $F$ of the force on the first particle should be proportional to its mass $m_1$ (e.g., if $m_1$ were 3 times as large, $F$ should also be 3 times as large). Similarly, the magnitude $F$ of the force on the second particle should be proportional to its mass $m_2$ (e.g., if $m_2$ were 3 times as large, $F$ should also be 3 times as large). If *both* $m_1$ and $m_2$ were 3 times as large, the magnitude $F$ of the force on either particle due to the other should then be larger by the factor $3 \times 3$. In other words, $F$ should be proportional to the *product* $m_1 m_2$ of the two masses.

Furthermore, observations of the motions of the planets suggest that $F$ depends on the distance R between the interacting particles so as to be proportional to $(1/R^2)$. [For example, if the distance $R$ were 3 times as large, $F$ would be multiplied by $(1/3)^2 = 1/9$.] The dependence of $F$ on the masses of the particles and on the distance $R$ between them can then be summarized by writing

$$F = G\frac{m_1 m_2}{R^2} \qquad \text{(A-2)}$$

where $G$ is a constant which does *not* depend on the masses of the particles or on the distance between them. Thus $G$ is a fundamental constant characterizing *all* gravitational interactions in our universe and called the "gravitational constant."

The numerical value of $G$ can be determined by careful experimental measurements of the kind discussed in Sec. D. The approximate value of $G$ is thus found to be

$$\boxed{G = 6.7 \times 10^{-11}\,\text{newton meter}^2/\text{kilogram}^2\,.} \qquad \text{(A-3)}$$

The units associated with $G$ are such as to assure the consistency of the units in the relation (A-2). In other words, the units in Eq. (A-3) assure that the force $F$ is properly expressed in terms of newton when the masses are expressed in terms of kilogram and the distance $R$ in terms of meter.





## DISCUSSION OF THE FORCE LAW

The preceding discussion about the direction and magnitude of the gravitational force can be summarized by this statement:

> *Gravitational force law:* Every particle exerts on every other particle a gravitational force $\vec{F}$ such that
>
> $\vec{F}$ is attractive; $F = Gm_1m_2/R^2$ (A-4)

This force law is called "Newton's law of gravitation" since it was first proposed by Isaac Newton in 1665. The validity of this law is supported by an impressive number of predictions well verified by observations of the kind discussed in the following sections.

Although the gravitational interaction exists between any two particles, Rule (A-4) indicates that the magnitude of this interaction depends crucially on the masses of the interacting particles. Indeed, since the gravitational constant $G$ is quite small, the gravitational force is almost immeasurably small when both interacting particles are common objects such as baseballs or cars (which have masses roughly between 1 kg and 1000 kg). Furthermore, the gravitational force is *utterly negligible* when both particles are atoms or molecules since these have exceedingly small masses. On the other hand, the gravitational force may be quite appreciable when at least one of the interacting particles has a large mass like that of a star or a planet (such as the earth). Hence the gravitational force becomes quite important when one considers the interaction of an ordinary object with the earth or the interaction between two astronomical bodies (such as stars or planets).

## GRAVITATIONAL INTERACTION BETWEEN LARGE OBJECTS

The gravitational force law, Rule (A-4), specifies the force on either of two interacting objects small enough to be considered as particles (so that each can be adequately described by the position of a single point). Hence this force law is *not* directly applicable if one (or both) of the interacting objects has a size which is not negligibly small compared to the distance between them.

How then can we find the gravitational force on a ball (or some other particle) due to a relatively large object such as the nearby earth? As

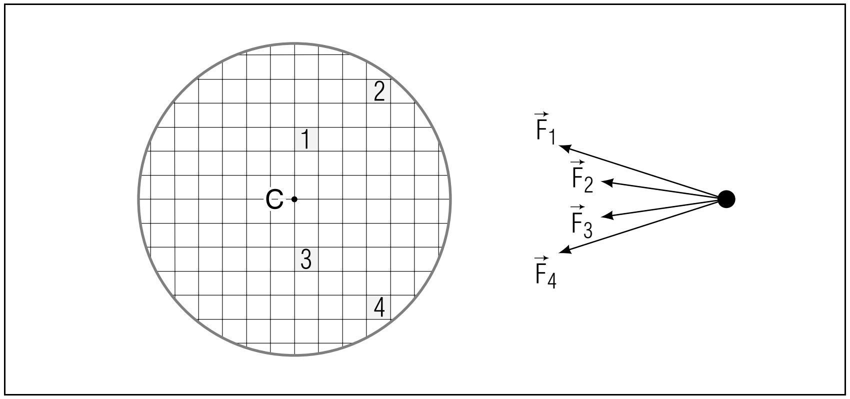



Fig. A-2: Gravitational force on a ball due to the many constituent particles in a large object (such as the earth).

illustrated in Fig. A-2, we need only think of the large object as consisting of many particles, each having a size much smaller than its distance from the ball. But the force on the ball due to each of the constituent particles of the large object can be found from the gravitational force law, Rule (A-4). The *total* force on the ball is then just the vector sum of all these individual forces. Although the actual calculation of such a vector sum may be laborious, it can be readily carried out in the simple case when the large object is spherically symmetrical, e.g., when it is a uniform spherical shell or a uniform sphere. This calculation leads to this simple result:*

> * The calculation, performed with the aid of some calculus, can be found in "The Gravitational Field Outside A Homogeneous Spherical Mass," MISN-0-109, available for credit in some courses: see your syllabus.

| A spherically symmetric object exerts on a particle outside this object the same gravitational force as if the entire object were concentrated in a particle at its center. | (A-5) |
|---|---|

This conclusion also implies that the gravitational force on a spherically symmetric object due to another outside spherically symmetric object is the same as if *each* object were concentrated at its center. *





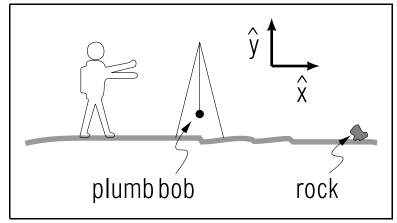



Fig. A-3.

> * The result, Rule (A-5), is not self-evident despite its simplicity, and is only true because the force between two particles is proportional to $1/R^2$.

For example, the resultant Rule (A-5) allows us to use information about the radius $R_0$ and mass $M$ of the earth to find the gravitational force $\vec{F}$ on a particle of mass $m$ located at the surface of the earth. Indeed, since the gravitational force $\vec{F}$ due to the earth is the same as that due to a single particle of mass $M$ located at the center of the earth, Rule (A-4) implies that $\vec{F} = GmM/R_0^2$ toward the center of the earth.

### Understanding the Gravitational Force Law (Cap. 1)

**A-1** *Example:* An astronaut surveying on the moon's surface determines the vertical direction by using a 0.8 kg lead weight (or "plumb bob") suspended from a string. (Fig. A-3). What is the gravitational force on the plumb bob due to a 100 kg rock which is small in size and located 20 meter from the bob? *(Answer: 103)*

**A-2** *Interpreting symbols:* The moon has a mass of $7 \times 10^{22}$ kg and a radius of $2\times10^6$ meter. If the plumb bob described in problem A-1 is suspended 0.5 meter above the moon's surface, what is the gravitational force on the bob due to the moon? Compare this force with your answer to problem A-1. Does the force due to the rock affect the astronaut's measurements significantly? *(Answer: 109) (Suggestion: [s-1])*

**A-3** *Properties:* (a) Describe the direction of the gravitational force on a particle $A$ due to a particle $B$. (b) Which of the following is a reasonable magnitude for the gravitational force due to the earth on a man at its surface? Which is a reasonable magnitude for the gravitational force on this man due to a woman 1 meter away? 7,000 newton; 700 newton; 70 newton; 0.7 newton; $7 \times 10^{-8}$ newton. *(Answer: 101)*

**A-4** *Comparing $F = Gm_1m_2/R^2$ with $F = mg$:* What are the values of the gravitational constant $G$ and the gravitational acceleration $g$ near the earth's surface? Which of the relations $F = mg$ (with the value of $g$ just stated) and $F = Gm_1m_2/R^2$ would you use to find each of these gravitational forces? (a) Force on the earth due to the sun. (b) Force on the moon due to the earth. (c) Force on a flying airplane due to the earth. *(Answer: 106)*

**A-5** *Relating quantities:* A 68 kg man jumps upward, along the unit vector $\hat{y}$. He is acted on by the downward gravitational force $-670$ newton$\hat{y}$ (since $(68\,\text{kg})(9.8\,\text{m/sec}^2) = 670$ newton). If the earth's radius is $6.4 \times 10^6$ meter, what is the mass of the earth? *(Answer: 110) (Suggestion: [s-4])*

### Relating Comparisons of Values (Cap. 2)

**A-6** A particle labeled "1" is acted on by a gravitational force of magnitude $F_0$ due to its interaction with a particle labeled "2." (a) What is the magnitude of the gravitational force on 2 due to 1? (b) Each particle is replaced by a new particle with a mass four times as large as the mass of the original particle. What then is the magnitude of the gravitational force on each particle due to the other? (c) Suppose these new particles move so that the distance between them is four times as large as the distance separating the original particles. What now is the magnitude of the gravitational force on each particle due to the other? *(Answer: 104) (Suggestion: [s-6])*

**A-7** (a) As a spacecraft travels from the earth to the moon, its distance from the earth's center increases from a value $(R_0)$ (while in orbit near the earth's surface) to a value $(60\,R_0)$ (as it reaches the moon). The corresponding values of the gravitational force on the spacecraft due to the earth are $F_e$ and $F'_e$. Compare these forces by writing the magnitude $F'_e$ as a number times $F_e$. (b) At one time during its trip to the moon, the spacecraft is at the same distance from the moon as it is from the earth. But the mass of the earth is 84 times the mass of the moon. Under these conditions, compare the magnitudes $F_e$ and $F_m$ of the gravitational forces on the craft due to the earth and moon by writing an expression for $F_m$ as a number times $F_e$ . *(Answer: 102) (Practice: [p-1])*





SECT.

## B RELATION BETWEEN $G$ AND $G$

Consider some particle (such as a ball or a spacecraft) which interacts only by gravity with some spherical astronomical body (such as the earth or the moon). Can we then use the gravitational force law, Rule (A-4), to predict the value of the gravitational acceleration $\vec{g}$ of the particle due to its interaction with the astronomical body? To answer this question, let us approach the problem systematically according to our strategy of text section D of Unit 409.

*Description*: Fig. B-1 illustrates the situation where the particle of mass $m$ is located at a distance $R$ from the center of the astronomical body of mass $M$. We want to find the acceleration $\vec{g}$ of the particle.

*Planning*: We can apply the equation of motion $m\vec{a} = \vec{F}$ to the particle. The only force on the particle is the gravitational force $\vec{F}_g$ due to the astronomical body. Furthermore, the acceleration $\vec{a}$ of the particle is called its gravitational acceleration $\vec{g}$ since the particle is only affected by gravity. Hence the equation of motion $m\vec{a} = \vec{F}$ is simply

$$m\vec{g} = \vec{F}_g \qquad \text{(B-1)}$$

According to Rule (A-5), the force $\vec{F}_g$ is the same as if the entire astronomical body were concentrated at its center $C$. By the gravitational force law, Rule (A-4), the direction of $\vec{F}_g$, and thus also of $\vec{g}$, is then *toward the center* of the astronomical body. Furthermore, the magnitude of $\vec{F}_g$ is $F_g = GmM/R^2$. TxtDescrImplementationThe equality of the *magnitudes* of both sides of Eq. (B-1) implies that $mg = F_g$. Hence

$$mg = G\frac{mM}{R^2}$$

Division of both sides of this equation by $m$ then yields this relationship between $G$ and $g$:

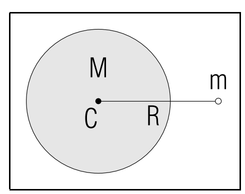



Fig. B-1: Gravitational interaction between a particle and an astronomical body.

$$\boxed{g = G\frac{M}{R^2}\,.} \qquad \text{(B-2)}$$

*Checking*: Our result Eq. (B-2) predicts properly the familiar fact that $g$ is independent of the mass or of all other properties of the particle. As expected, it also predicts that $g$ decreases with increasing distance $R$ from the astronomical body.

## DISCUSSION

The result Eq. (B-2) shows how the magnitude $g$ of the gravitational acceleration of a particle is related to the gravitational constant $G$, to the mass $M$ of the astronomical body producing this acceleration, and to the distance $R$ of the particle from the *center* of the astronomical body. For example, if the particle is at the surface of an astronomical body, the distance $R$ is merely the radius $R_0$ of this body. Hence we can use the known value of the gravitational constant $G$ to calculate the gravitational acceleration $\vec{g}$ near the surface of any astronomical body (such as the earth or the moon) if we know the mass $M$ and radius $R_0$ of this body. Correspondingly, we can then also find the weight $mg$ of any object of mass $m$ at the surface of this body.

Actually, the mass of the earth is not directly known since there is no practical way of comparing it directly with the standard kilogram. But, if we know the gravitational constant $G$, we can use Eq. (B-2) to calculate the mass $M$ of the earth from its known radius $R$ and from the measured value of $g$ at the surface of the earth (See Problem

B-2 *KNOWING ABOUT GRAVITATIONAL ACCELERATION:*

B-1 Which of the following phrases correctly describes the gravitational acceleration $g_m$ at the moon's surface? (a) The acceleration of the moon relative to the earth. (b) The acceleration (relative to the moon) of a rock dropped at the moon's surface and so moving subject only to gravitational interaction with the moon. (c) The acceleration (relative to the moon) of a satellite in orbit just above the moon's surface. *(Answer: 107)*





**Relating Motion to Gravitational Interaction (Cap. 3)**

B-2 Use these measured values to find the mass of the earth: Gravitational acceleration near the earth's surface is $10\,\mathrm{m/s^2}$, the earth's circumference is $4.0 \times 10^7$ meter. Begin your work with the equation of motion ($m\vec{a} = \vec{F}$) and follow the reasoning leading to Eq. (B-2). NOTE: See Eq. (A-3) for the value of $G$. *(Answer: 111)*

SECT.

# C PLANETARY MOTION

Let us now apply the gravitational force law to investigate the motion of a particle around an astronomical body. To be specific, we shall discuss the motion of a planet (such as the earth) around the sun, although the same argument is applicable to the motion of the moon or of an artificial satellite around the earth. Since the mass $M$ of the sun is much larger than the mass $m$ of the planet, the acceleration of the sun is negligibly small compared to that of the planet. Hence the sun can, to excellent approximation, be regarded as fixed relative to some inertial frame.

We shall assume that the planet moves around the center of the sun with some constant speed $v$ in a circular orbit of radius $R$. (In fact, all the planets do move in orbits which are nearly circular.) Can we then use the gravitational force law, Rule (A-4), to predict the speed $v$ of the planet or the time $T$ it requires to go once around the sun?

To answer this question, we shall again approach the problem systematically.

*Description*: Fig. C-1 illustrates the situation. We assume that we know the mass $m$ of the planet, the mass $M$ of the sun, the radius $R$ of the circular orbit, and the fact that the speed $v$ of the planet is constant. We should like to find the value of this speed.

*Planning*: We can apply the equation of motion $m\vec{a} = \vec{F}$ to the planet. Since the only force acting on the planet is the gravitational force $\vec{F}_g$ due to the sun, we can write

$$m\vec{a} = \vec{F}_g \qquad \text{(C-1)}$$

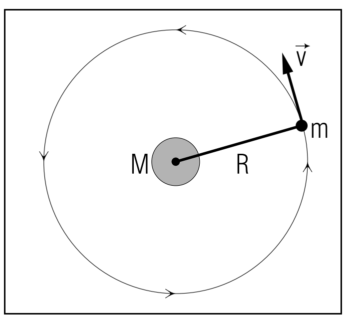



Fig. C-1: Motion of a planet in a circular orbit around the sun.





Since the gravitational force is attractive, $\vec{F}_g$ is directed toward the sun, i.e., toward the center of the circular orbit of the planet. Because the planet travels around this circular orbit with constant speed, we also know from text section 2c of Unit 376 that the acceleration $\vec{a}$ of the planet is directed toward the center of the circular orbit. Hence the direction of a is properly the same as that of $\vec{F}_g$, as required by the equation of motion Eq. (C-1). Thus the only additional requirement of Eq. (C-1) is that the *magnitudes* of both sides of this equation are equal, i.e., that

$$ma = F_g \qquad \text{(C-2)}$$

Here we know, by the gravitational force law ,Rule (A-4), that $F_g = GmM/R^2$. Furthermore we know, from our discussion of circular motion in Relation (4-,) of Unit 376 that $a = v^2/R$.

*Implementation*: By substituting the preceding expressions for $F_g$ and a into Eq. (C-2), we obtain

$$m\frac{v^2}{R} = G\frac{mM}{R^2} \qquad \text{(C-3)}$$

Division of both sides of this equation by $m$ then yields

$$\frac{v^2}{R} = G\frac{M}{R^2}$$

so that:

$$\boxed{v^2 = \frac{GM}{R} \quad \text{or} \quad v = \sqrt{\frac{GM}{R}}} \qquad \text{(C-4)}$$

*Checking*: Our result shows that the speed $v$ of the planet is independent of its mass $m$ or other properties [because the same mass $m$ appeared on both sides of Eq. (C-3) and thus "cancelled"]. This conclusion is expected since any particle subject only to gravitational interaction moves in a manner independent of its mass or other properties.

## DISCUSSION

The result Eq. (C-4) allows us to use a knowledge of the gravitational constant $G$ to find the speed $v$ of any planet if we know the mass $M$ of the sun and the radius $R$ of the planet's orbit. (Conversely, we can use measurements of the speed $v$ and orbital radius $R$ of any one planet to find the mass $M$ of the sun.) In practice, the speed $v$ of a planet is determined most readily by measuring its period $T$ (i.e., the time required for the planet to go once around its orbit). Indeed, the speed $v$ is just the circumference of the orbit divided by $T$ [i.e., $v = 2\pi R/T$, as pointed out in statement (F-8) of Unit 406. This connection between $v$ and $T$ can be used to find a direct relation between the period $T$ of a satellite and the radius $R$ of its orbit. (See Problem

F-1 *T:* R was historically quite important and is known as "Kepler's third law of planetary motion."

### Knowing About Motion due to the Gravitational Force

C-1 A "lunar excursion module" (LEM) and its command module both move separately around the moon each in a circular orbit of radius $2 \times 10^6$ meter. The mass of the command module is three times as large as the mass of the LEM, and each vehicle interacts only with the moon. To dock the LEM with the command module, the pilot must know the speeds $v_L$ and $v_C$ of these objects relative to the moon. Express the speed $v_L$ as a number times $v_C$. *(Answer: 114) (Suggestion: [s-2])*

### Relating Motion to Gravitational Interaction (Cap. 3)

Demonstrating this capability requires beginning with the equation of motion ($m\vec{a} = \vec{F}$) to systematically solve problems like these:

C-2 In a period of 1 year $= 3.15\times10^7$ sec, the earth moves once around the sun in a circular orbit of radius $1.5\times10^{11}$ meter. Thus relative to the solar frame, the earth's speed is $2\pi(1.5\times10^{11}$ meter$)/(3.15\times10^7$ sec$)$ $= 3.0 \times 10^4$ m/s. Use the fact that the earth interacts appreciably only with the sun to express the sun's mass in terms of symbols for known quantities. Then find the value of this mass. *(Answer: 117) (Suggestion: [s-5])*

C-3 According to its planned path, a space-observation probe should pass Jupiter at a distance of $2 \times 10^5$ km from this planet's center. (km $= 10^3$ meter) When the probe is at this distance from the planet, it interacts appreciably only with Jupiter which has a mass of $2 \times 10^{27}$ kg. At this time, what is the probe's acceleration (relative to the solar frame)? *(Answer: 105) (Practice: [p-2])*





### Relating Comparisons of Values (Cap. 2)

C-4 A spacecraft, interacting only with the earth, travels in a circular orbit. The craft's rockets are then briefly fired so that thereafter it moves in a different circular orbit with a radius smaller than the radius of the original orbit. According to Eq. (C-4), is the final speed of the spacecraft larger or smaller than its original speed? *(Answer: 108)*

SECT.

# D MEASURING $G$

The magnitude of the gravitational force specified by the force law, Rule (A-4), depends on the gravitational constant $G$, a fundamental constant characteristic of our universe. As we have seen, a knowledge of the value of $G$ permits us to calculate gravitational forces and also to use experimental observations to deduce important astronomical information (e.g., to find the mass of the earth or of the sun).

How can one actually determine the value of $G$? In principle, it is only necessary to measure the magnitude $F$ of the gravitational force on one known particle (or spherically symmetric object) due to another. Indeed, if one knows the masses $m_1$ and $m_2$ of these particles and the distance $R$ between them, one can use the gravitational force law $F = Gm_1m_2/R^2$ to find the value of $G$ from the measured value of $F$.

The difficulty is that the actual masses of astronomical objects (such as the earth or the sun) are not directly known since there is no direct way of comparing the masses of such large objects with the standard kilogram stored near Paris. Hence it is necessary to carry out experimental measurements of $G$ by using objects of everyday size. But the masses of such objects are so small that the gravitational force on one such object due to another is extremely small and thus very difficult to measure. A sufficiently sensitive technique for measuring such extremely small forces allows one, however, to achieve these two aims: (1) One can directly demonstrate that the gravitational interaction exists also between any two objects of everyday size. (2) One can measure the value of $G$. Henry Cavendish (1731-1810) first performed such measurements in 1798 (almost a century after Newton) by measuring the small gravitational force between two lead spheres of known masses. He used a very thin vertical wire supported from its top and attached at its bottom to the midpoint A of a horizontal rod connecting two identical lead spheres (labeled by 1 in Fig. D-1). This arrangement is called a Cavendish Experiment

Suppose now that two other identical spheres (labeled by 2) are brought close to the spheres 1 (as shown in Fig. D-1). Then each sphere 1 experiences a small horizontal gravitational force due to the adjacent sphere 2 and thus moves toward it. The result is that the wire is twisted through some small angle $\theta$ which can be measured (by observing light reflected from a mirror attached to the wire). If one knows from prior





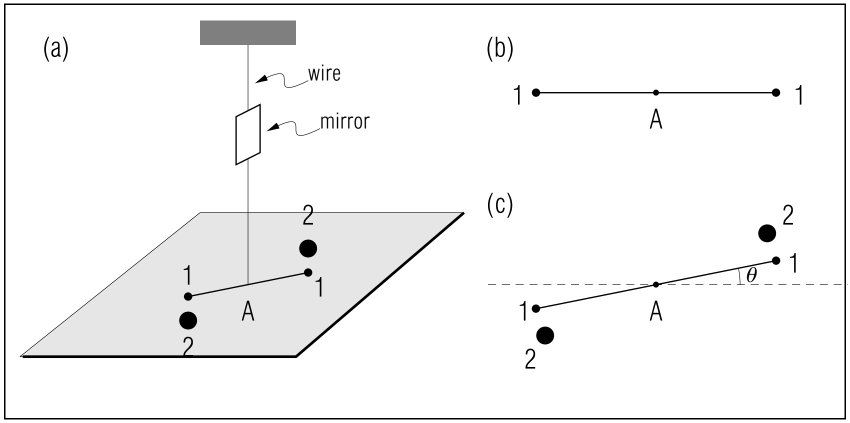



Fig. D-1: Cavendish experiment for the determination of $G$. (a) Perspective view of the experimental arrangement. (b) Top view of the lead spheres 1 originally at rest. (c) Top view of the lead spheres 1 finally at rest in the presence of the lead spheres 2.

measurements how large a twist of the wire is produced by *known* forces (such as forces applied by spring), the angle $\theta$ measured in the experiment provides direct information about the magnitude of the gravitational force on sphere 1 due to sphere 2. Since the distance between the centers of the spheres can also be measured, one has then all the information necessary to determine the gravitational constant $G$. According to the most recent measurements, the value of $G$ is now known to be

$$G = (6.6732 \pm 0.0031) \times 10^{-11}\,\text{newton meter}^2/\text{kg}^2 \qquad \text{(D-1)}$$

SECT.
# E SUMMARY

## IMPORTANT RESULTS

Gravitational force law: Rule (A-4)

$\vec{F}$ is attractive; $F = G m_1 m_2 / R^2$

Gravitational force due to a spherically symmetric object: Rule (A-5)

Force on a particle outside the object is the same as if the entire object were concentrated at its center.

## USEFUL KNOWLEDGE

Definition of gravitational acceleration $\vec{g}$: (Sec. B)

Motion of object, due to gravity, does not depend on object's mass: (Sec. C)

## NEW CAPABILITIES

You should have acquired the ability to:

(1) Understand the gravitational force law. (Sec. A)

(2) For a relation (such as the gravitational force law) which involves only multiplication and division, use a comparison of the values of one quantity to compare the corresponding values of another quantity. (Secs. A, B, C, [p-1])

(3) If a particle is acted on only by the gravitational force due to an astronomical body, relate: the acceleration of the particle (or its speed along a circular orbit), its distance from the astronomical body, and this body's mass. (Secs. B and C, [p-2])

### Organization of Relations

**E-1** (a) Use the relation $F = G m_1 m_2 / R^2$ to find the weight of a 64 kg man at the earth's surface. The earth has a mass of $6.0 \times 10^{24}$ kg and a radius of $6.4 \times 10^6$ meter. (b) Then find the same man's weight using the relation $F = mg$, where $g = 9.8\,\mathrm{m/s}$ (precise to two significant figures). (c) Are the two results the same? *(Answer: 112)*





SECT.
# F PROBLEMS

### Predicting Motion In Gravitational Force

Use $m\vec{a} = \vec{F}$ to begin the solution of each of these problems:

**F-1** *Kepler's third law:* As discussed in Sec. C, we can use the gravitational force law to find the speed $v$ of any planet traveling in a circular orbit around the sun. Historically, however, it was easier to measure a planet's period $T$ than its speed. Relate a planet's period to its speed, and use this relation to show that the quantity $R^3/T^2$ is equal to a quantity which is the same for all planets moving around the sun. According to this relation (called Kepler's third law), does a planet which is farther from the sun have a period which is *longer* or *shorter* than that of a planet which is nearer to the sun? *(Answer: 115) (Suggestion: [s-3])*

**F-2** *Distance to the moon:* We can calculate the distance $R$ between the centers of the earth and moon from quantities measurable on earth. Consider the moon as a particle moving with constant speed in a circular path relative to the inertial frame of the earth. (a) Find an expression for $R$ in terms of the period $T$ of the moon's revolution, the mass $M$ of the earth, and known quantities. (b) In this expression the only quantity which we cannot measure is the mass $M$ of the earth. Apply the equation of motion to a particle near the earth's surface in order to express $M$ in terms of the circumference $C$ of the earth and the magnitude $g$ of the gravitational acceleration near the earth. (c) Combine your results to express $R$ in terms of these quantities measurable on earth: the circumference $C$ of the earth, the gravitational acceleration $g$, period $T$ of the moon's motion. (d) Use the following values to find the distance $R$ between the center of the earth and the moon: $C = 40,000\,\text{km}$, $T = 28\,\text{day}$ $= 2 \times 10^6\,\text{sec}$. *(Answer: 116)*

**F-3** *Orbit for a communications satellite:* A communications satellite is placed in a "synchronous" orbit, so that it remains always above the same point on the earth's surface. Thus this satellite moves once around its orbit in a time equal to the time required for one rotation of the earth, i.e., in a time of $24\,\text{hour} = 9 \times 10^4\,\text{sec}$. What is the radius of this satellite's orbit? (The satellite can be considered as a particle interacting only with the earth, and moving relative to an inertial reference frame in which the earth's center is at rest.) *(Answer: 113)*

# PRACTICE PROBLEMS

**p-1** *RELATING COMPARISONS OF VALUES (CAP. 2):* Astronomers estimate that the mass of the planet Pluto is about the same as the mass of the earth. However, the distance from the sun to Pluto is about 40 times as large as the distance from the sun to the earth. Compare the gravitational forces on these two planets due to the sun by writing an expression for the magnitude $F_p$ of the force on Pluto in terms of the magnitude $F_e$ of the force on the earth. *(Answer: 4) Now: Return to text problem A-6 and make sure your work is correct.)*

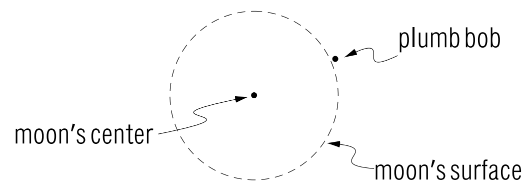
**p-2** *RELATING MOTION TO GRAVITATIONAL INTERACTION (CAP. 3):* The moon has a mass of $7 \times 10^{22}$ kg and a radius of $1.7 \times 10^6$ meter. An observation satellite moves in a circular orbit at a height of $8 \times 10^5$ meter above the moon's surface. What is the speed of this satellite (relative to the moon)? Use the approximate value $G = 7 \times 10^{-11}$ newton meter$^2$/kg$^2$. *(Answer: 2) (Suggestion: review text problems C-2 and C-3.)*





# SUGGESTIONS

**s-1** *(Text problem A-2):* When applying the relation $F = Gm_1m_2/R^2$ to a spherically symmetric object such as the moon, the distance $R$ is measured from the center of the object.




Here $R = 2 \times 10^6$ meter + 0.5 meter. But because 0.5 meter is negligibly small compared with $2 \times 10^6$ meter, $R = 2 \times 10^6$ meter.

**s-2** *(Text problem C-1):* Review the argument leading to equation (C-4). This argument applies to any particle of mass $m$ in a circular orbit around an astronomical object of mass $M$. Notice that the speed $v$ of such a particle does not depend on its mass $m$. Because the LEM and the command module move in circular orbits with the same radius, they have the same speed, even though their masses are different.

**s-3** *(Text problem F-1):* Your solution should begin with the equation of motion $m\vec{a} = \vec{F}$ for a planet of mass $m$. Then the planet's acceleration has a magnitude $a = v^2/R$, where $v = 2\pi R/T$ is the planet's constant speed as it moves around a circular orbit of radius $R$ in a time $T$. (See text section F of Unit 406)

**s-4** *(Text problem A-5):* Note that we know the direction of $\vec{F}$ from the preceding problem, problem A-4.

The equation $F = Gm_1m_2/R^2$ relates magnitudes. Thus $F$ is a positive number, without a sign or a direction.

To simplify arithmetic, first write an algebraic expression for the desired quantity in terms of symbols for known quantities. Then substitute values, and use convenient relations such as $670/6.7 = 100$ before performing further arithmetic.

**s-5** *(Text problem C-2):* Begin with the equation of motion for the. earth, $m\vec{a} = \vec{F}$. Because the earth interacts appreciably only with the sun, the magnitude of the total force $\vec{F}$ on the earth is just $F = GMm/R^2$, where $M$ is the mass of the sun and $R$ the distance between the earth and sun. Because the earth moves with constant speed along a circular path, its acceleration has the magnitude $a = v^2/R$, where $v$ is the speed of the earth.

**s-6** *(Text problem A-6):* In using a relation to compare values, it is helpful to group separately quantities with values which remain the same and quantities with values which change. For example, the original value of $F$ is $F_0 = Gm_1m_2/R^2$, where $m_1$ and $m_2$ have their original values. To answer the question in part (b), we write the gravitational force law in this way:

$$F = G\frac{m_1m_2}{R^2} = \left(\frac{G}{R^2}\right) m_1m_2$$

where $(G/R^2)$ remains the same, but $m_1$ and $m_2$ have differing values.

If particle 1 *only* is replaced by a particle with mass *twice* as large as that of the original particle 1, is the new value for $F$ *twice as large*, *one-half as large*, *four times as large*, or *one-fourth as large* as the original value $F_0$?

► ____________________

If both particles 1 and 2 are replaced by new particles with masses twice as large as the masses of the original values, is the new value of $F$ *twice as large, one-half as large, four times as large*, or *one-fourth as large* as the original value $F_0$?

► ____________________

*(Answer: 3) Now: If right, go to [p-1]. If wrong, or if you need further help, go to [s-7].*

**s-7** *Suggestion [s-6]):* Part (b): Let us choose some sample values for the masses of the two particles and the distance between them. For example, suppose the particles each have a mass of 1 kg, and are separated by a distance of 1 meter. For simplicity, throughout this frame use the value $G = 7 \times 10^{-11}$ newton meter$^2$/kg$^2$.

What is the magnitude $F_0$ of the force on each particle due to the other?





► $F_0 =$ ____________________

Now suppose that each particle is replaced by a new particle with a mass of 4 kg (four times as large a mass as the original particles).

What now is the magnitude of the force on each particle due to the other?

► ____________________

Express your preceding answer as a number times $F_0$.

► ( ____________________ )$F_0$

Part (c): Finally suppose that the new particles, each of mass 4 kg, are separated by a distance of 4 meter (a distance four times as large as their original separation).

What now is the magnitude of the force on each particle due to the other?

► ____________________

Express your preceding answer as a number times $F_0$.

► ( ____________________ )$F_0$

*(Answer: 1) Now: Go to [p-1].*

# ANSWERS TO PROBLEMS

1. $F_0 = 7 \times 10^{-11}$ newton, $112 \times 10^{-11}$ newton, $16F_0$, $7 \times 10^{-11}$ newton, (1) $F_0$
2. $1 \times 10^3$ m/s
3. twice as large, four times as large
4. $F_p = (1/1600)F_e = (6.2 \text{ or } 6.3) \times 10^{-4} F_e$

101. a. Directed from A towards B, i.e., it is an attractive force.
     b. 700 newton, $7 \times 10^{-8}$ newton
102. a. $F'_e = (1/3600)F_e = (2.8 \times 10^{-4})F_e$
     b. $F_m = (1/84)F_e = (0.012)F_e$
103. $1 \times 10^{-11}$ newton$\hat{x}$
104. a. $F_0$
     b. $16F_0$
     c. $F_0$
105. 3 m/s$^2$ towards Jupiter
106. $G = 6.7 \times 10^{-11}$ newton meter$^2$/kg$^2$, $g = 10$ meter/sec$^2$, $F = Gm_1m_2/R^2$ applies to all of these forces. $F = mg$ applies only near the earth's surface, i.e., for (c).
107. (b) and (c)
108. larger
109. $-(0.9 \text{ or } 1)$ newton$\hat{y}$ No. Force due to rock is completely negligible.
110. $6.0 \times 10^{24}$ kg
111. $M = gC^2/(4\pi^2 G) = 6.0 \times 10^{24}$ kg; $C$= earth's circumference
112. a. $6.3 \times 10^2$ newton
     b. $6.3 \times 10^2$ newton
     c. yes
113. $\sqrt[3]{81 \times 10^{21} \text{ meter}^3} = (4 \text{ or } 5) \times 10^7$ m $= (4 \text{ or } 5) \times 10^4$ km





114. $v_L = (1)v_C = v_C$
115. $R^3/T^2 = GM/(4\pi^2)$, where $M$ is the sun's mass. Longer.
116. a. $R = \sqrt[3]{GMT^2/(4\pi^2)}$
     b. $M = gC^2/(4\pi^2 G)$
     c. $R = \sqrt[3]{C^2 g T^2/(16\pi^4)}$
     d. $R = \sqrt[3]{40 \times 10^{24} \text{ meter}^3} = (3 \text{ or } 4) \times 10^8$ meter $= (3 \text{ or } 4) \times 10^5$ km
117. $M = v^2R/G$, where $v$ is the earth's speed and $R$ the radius of its orbit. $M = 2.0 \times 10^{30}$ kg.

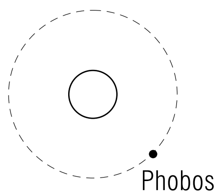
# MODEL EXAM

## USEFUL INFORMATION

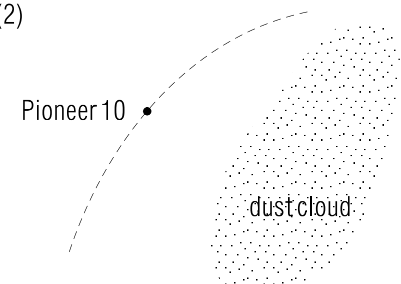
$$G = 6.7 \times 10^{-11}\,\text{newton meter}^2/\text{kilogram}^2\,.$$

1. **Using two expressions for gravitational force.** Both of the relations $F = mg$ and $F = Gm_1m_2/R^2$ describe the gravitational force. For each of the following questions, suppose that you know just the masses of the two interacting objects and the distance between their centers, and that you will use the value $g = 10\,\text{m/s}^2$. Then write $F = Gm_1m_2/R^2$, $F = mg$, neither, or both to indicate the relation(s) which you could use to find the indicated force.

(1)




(2)




The satellite Phobos of the planet Mars travels in a circular orbit of radius 9,000 km while the radius of Mars is 3,500 km. Thus the preceding drawing (1) is approximately drawn to scale.

a. What relation(s) can be used to find the magnitude of the gravitational force on Phobos due to Mars?

The Pioneer 10 interstellar observation probe approaches the large dust cloud indicated in the preceding drawing (2).

b. What relation(s) can be used to find the magnitude of the gravitational force on Pioneer 10 due to the dust cloud?

A man on earth flips a dime to settle an argument.





c. Which relations can be used to find the magnitude of the gravitational force on the dime due to the earth?

2. **Forces on the comet Kohoutek.** Shortly after its discovery in March 1973, comet Kohoutek was observed at a distance of $50 \times 10^7$ km from the sun. On December 28, 1973 the comet was $2 \times 10^7$ km from the sun. Compare the magnitudes $F_M$ and $F_D$ of the gravitational force on the comet due to the sun in March and on December 28. (Neglect any change in the comet's mass during this time.)

   Express $F_M$ as a number times $F_D$.

3. **Gravitational force due to the planet Venus.** The planet Venus has a radius of $6 \times 10^6$ meter (approximately the same as the earth's radius), but its mass is only 2/3 as large as the mass of the earth, or about $4 \times 10^{24}$ kg.

   a. An astronaut (with his equipment) has a mass of 120 kg. What would be the gravitational force on this astronaut (due to Venus) if he stood on the surface of Venus?

   b. What is the gravitational force on this astronaut due to the *earth* as he stands on the *earth's* surface? The radius of the earth is $6.4 \times 10^6$ m, and its mass is $6.0 \times 10^{24}$ kg.

**Brief Answers**:

1. a. $F = Gm_1m_2/R^2$
   b. Neither
   c. Both
2. $F_M = (1/625 \text{ or } 1.6 \times 10^{-3})F_D$
3. a. $9 \times 10^2$ newton
   b. $1.2 \times 10^3$ newton