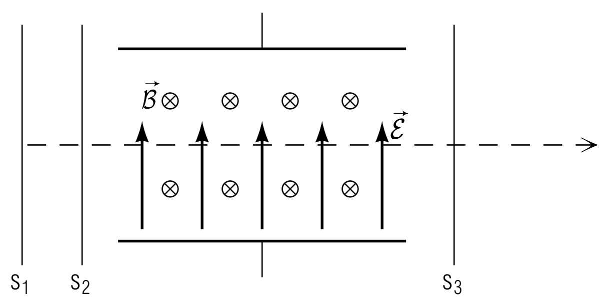
MISN-0-310

THE NATURE OF ATOMS AND ELECTRONS: THE MILLIKAN, THOMSON, AND RUTHERFORD EXPERIMENTS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

THE NATURE OF ATOMS AND ELECTRONS:
THE MILLIKAN, THOMSON, AND RUTHERFORD EXPERIMENTS

by
J. H. Hetherington

Title: **The Nature of Atoms and Electrons: the Millikan, Thomson, and Rutherford Experiments**

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Input Skills:

1. Calculate the electric field within a parallel plate capacitor given the potential difference (MISN-0-134).
2. Calculate the force on a charged particle in a given electric field (MISN-0-115).
3. Calculate the force on a charged particle moving in a magnetic field (MISN-0-122).
4. Helpful; Calculate the orbits of particles in inverse square force fields (MISN-0-105) or (MISN-0-106).

Output Skills (Knowledge):

K1. Describe and derive the equations which relate the quantity to be determined to the quantities which are experi mentally controlled in: (i) the Millikan oil-drop experiment; (ii) Thomson's determination of e/m; and (iii) the Rutherford scattering experiment.

Output Skills (Problem Solving):

S1. Calculate the deflection of charged particles passing through given electric and magnetic fields.

S2. Calculate the differential cross section for scattering of given charged particles by nuclei of given charge.

External Resources (Required):

1. Henry Semat, *Introduction to Atomic and Nuclear Physics*, Third Ed., Rinehart, New York (1954). For access, see this module's *Local Guide*.
2. R.T. Weidner and R.L. Sells, *Elementary Modern Physics*, alt. 2nd ed., Allyn and Bacon, Boston (1973). For access, see this module's *Local Guide*.

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1. Introduction

At the forefront of physics there is always an effort to describe qualitatively the nature of matter on the finest scale we can measure. The three experiments studied in this unit were important "forefront" experiments of the early part of this century. They formed the original basis for our understanding of the nature of atoms and electrons. From these experiments we find that atoms have a small dense nucleus, that the electrons which surround the nucleus have a very small mass compared with the nucleus, and that the fundamental unit of charge is the charge on the electron. These ideas are now part of our general knowledge as reasonably educated members of the 20th Century, yet they were first implied by these relatively recent experiments. One of the goals of a study of physics is to give an understanding of the reasons behind the pictures we have of the microscopic structure of matter. The objectives of this unit are intended to contribute to this goal.

2. Study Program

HS:[1] Study Section 2-4, pp. 35-38 (Output Skill K1) and Section 2-6 (Output Skill K2).

WSM:[2] Study Section 6-1 (Output Skill K3).

Note that Eq. 6-7, if written without the proportionality sign, is:

$$\frac{dN_s}{N_i} = \frac{1}{16}\left(\frac{k_e Q_1 Q_2}{E_k}\right)^2 \frac{nt}{\sin^4(\theta/2)} d\Omega\,, \qquad (1)$$

[1]Henry Semat, *Introduction to Atomic and Nuclear Physics*, Third Ed., Rinehart, New York (1954). For access, see this module's *Local Guide*.

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and the equation below 6-7 is (using Z_1 and Z_2 for the atomic number of the target and projectile):

$$\frac{dN_s}{N_i} = \left[\frac{Z_1^2 Z_2^2 k_e^2 e^4}{16 E_k^2 \sin^4(\theta/2)}\right] nt d\Omega\,. \qquad (2)$$

Note that the quantity in square brackets is called "differential cross section" which is written $d\sigma/d\Omega$. It contains information about the physics of the scattering process. The factors outside the square bracket contain information about the experimental set-up (density and thickness of foil, size and distance of counter, $d\Omega = A/r^2$ where A is the area of counter and r is distance from target to counter). Also note that Semat uses mixed cgs units in calculations and quotes electric charge in e.s.u. You should compare Semat's electrostatic and magnetic formulas with the MKS version provided in Section 1-3 of WSM to see if there is any conflict.

Acknowledgments

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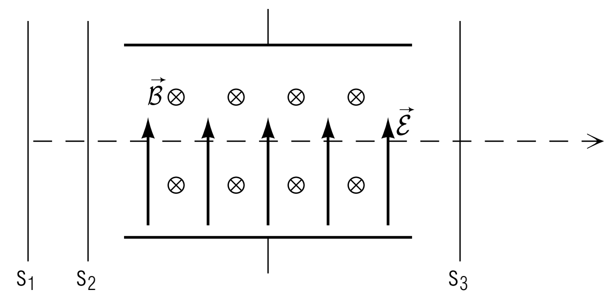
LOCAL GUIDE

The readings for this unit are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them as "The readings for CBI Unit 310." Do **not** ask for them by book title.

PROBLEM SUPPLEMENT

1. An electron leaves a heated filament and is accelerated by a +1500 V anode with a hole in it. What is its kinetic energy and velocity? Is it relativistic? After the acceleration it passes through a deflection plate arrangement. The plates are 5 cm long and have a potential difference of 100 V, and are separated by 1 cm. What is the vector velocity of the electron after passing through the plates? What is the ratio of the velocity of the electron parallel to and perpendicular to the undeflected electron direction? Does this ratio depend on e/m?

2. A velocity separator is made with a crossed magnetic and electric field:



   The slits define a beam of particles which passes straight through. Calculate the velocity of an undeflected electron in terms of the electric field E and the magnetic field B. If a voltage of 100 V is applied to a 1 cm deflection plate gap, what is the field required to let electrons of 2×10^7 m/s pass through.

3. A beam of protons ($q = e$) of 1 MeV energy bombards an iron foil ($Z = 26$). What is the differential cross-section for this scattering as a function of angle? Put in factors to obtain proper magnitude as well as angular dependence. How close does the proton come to the iron nucleus in a head-on collision?

4. In problem 3 above, if the iron foil is 0.01 cm thick (assume iron has a density of 7.5 mg/cm^3 and an atomic weight of 56), and if the beam of protons has 10^{10} protons/s, how many protons will be scattered into a counter subtending 0.1 steradians at 45° (4π steradians is the solid angle of all directions combined)?

Brief Answers:

1. Initial velocity of electron on ejection from filament can be neglected. The kinetic energy of the electron is 1500 eV after passing the anode, or, in MKS units:

   $E = eV = (1.6 \times 10^{-19}\,\mathrm{C})(1500\,\mathrm{V}) = 2.4 \times 10^{-16}\,\mathrm{J}\,.$

   where E is the Kinetic Energy.

   The velocity is:

   $$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19}\,\mathrm{C} \times 1500\,\mathrm{V}}{9.106 \times 10^{-31}\,\mathrm{kg}}} = 2.2 \times 10^{7}\,\mathrm{m/s}$$

   Time between plates is $T = \ell/v$ where ℓ is the length of the plates.

   $$a_\perp = \frac{Ee}{m} = \frac{V_D}{d} \cdot \frac{e}{m}$$
   $$v_\perp = a_\perp T = \frac{V_D}{d}\frac{e}{m}\frac{\ell}{v} = \frac{V_D}{d}\frac{e}{m}\frac{\ell}{\sqrt{2eV/m}} = \frac{V_D}{\sqrt{V}}\frac{\ell}{d}\sqrt{\frac{e}{m}}\frac{1}{\sqrt{2}} = 3.83 \times 10^{6}\,\mathrm{m/s}$$
   $$\vec{v} = (v_\perp, v) = (3.83 \times 10^{6},\ 2.2 \times 10^{7})$$

   $$\text{ratio} = \frac{v_\perp}{v} = \frac{1}{\sqrt{V}}\frac{1}{\sqrt{e/m}}\frac{1}{\sqrt{2}}\frac{V_D}{\sqrt{V}}\frac{\ell}{d}\sqrt{\frac{e}{m}}\frac{1}{\sqrt{2}} = \frac{V_D}{2V}\frac{\ell}{d} = 0.1667$$

   Notice that, in the end, the ratio is independent of e/m.

2. $0 = F = e(E + \vec{v} \times \vec{B})$ so $E = vB\,,$

   where we presume $\vec{B}$ is perpendicular to $\vec{v}$ and that the directions are such that the magnetic field deflects the particle in a direction opposite to the direction the electric field deflects it.

   Then: $v = E/B$.

   $$B = \frac{E}{v} = \frac{100 \times 100\,\mathrm{V/m}}{2 \times 10^{7}\,\mathrm{m/s}} = 0.5 \times 10^{-3}\,\mathrm{W/m^2}$$

3. Note that the charge on the proton is only half the charge on the α-particle and see eq. 13.28, AM.

$$\frac{d\sigma}{d\Omega} = \frac{1}{16}\left(\frac{k_e Z e^2}{T_0}\right)^2 \frac{1}{\sin^4(\theta/2)}$$
$$= \frac{1}{16}\left(\frac{26(1.6 \times 10^{-19})^2 8.99 \times 10^{9}\,\mathrm{Nm^2/C^2}}{10^{6} \times (1.6 \times 10^{-19})\,\mathrm{Nm}}\right)^2 \frac{1}{\sin^4(\theta/2)}$$
$$= 8.74 \times 10^{-29} \frac{1}{\sin^4(\theta/2)}\,\mathrm{m^2}$$

4. $$\frac{\#}{sec} = \frac{d\sigma}{d\Omega}\frac{\#}{\mathrm{m}^2}\,\mu\,\Delta\Omega$$

   where # of atoms/m^2 in the foil $= n_0 = t \cdot \rho = 10^{-4}\rho$ and $\Delta\Omega = 0.1$ ster.

   and $\rho =$ density $= (7.5\,\mathrm{gm/cm^3})\left(\frac{6.023 \times 10^{23}}{56}\,\mathrm{atoms/gm}\right)(10^{6}\,\mathrm{cm^3/\,m^3})$

   and $\mu =$ # of particles/sec in the beam $= 10^{10}$.

   Then: $n_0 = 0.807 \times 10^{25}\,.$

   $$\frac{d\sigma}{d\Omega} = 8.74 \times 10^{-23}\frac{1}{(\sin 22.5^\circ)^4} = 4.07 \times 10^{-25}\,\mathrm{m^2}$$

   Finally: $\dfrac{\#}{\mathrm{sec}} = 3.28 \times 10^{7}$.

   At nearest approach in head-on collision potential energy equals original kinetic energy.

   KE $= Ve$ where $V = 10^{6}\,\mathrm{V}$ (V is now Volts) and e is the electron charge.

   $V = \dfrac{26\,e^2}{r}\,k_e$ [See Eq. (1-5) W.S.]

   $$r = 26\,e^2\,k_e\frac{1}{Ve} = k_e\,26e/V = (8.99 \times 10^{9})\frac{26(1.6 \times 10^{-19})}{10^{6}}\frac{\mathrm{N/m^2}}{CV} = 3.74 \times 10^{-14}\,\mathrm{m}$$

   How does this compare with the radius of a nucleus?

MODEL EXAM

1. One of Output Skills K1-K4, with the caveat that only 50 words or only a few lines be used in the explanation.

2. A certain particle has a differential cross-section for scattering from a certain kind of atom at 50° of $10^{-26}\,\text{m}^2/\,\text{stear}$. How many particles are scattered into a counter centered at 50° from the beam if the counter has an area of $10\,\text{cm}^2$ and is 30 cm from the target. The beam has 10^5 particles per second. The target material has an atomic density of $10^{27}\,\text{atoms/m}^3$ and a thickness of 10^{-3} meters.

3. An electron in a cathode ray tube is accelerated through a potential of 3000 V. It is then deflected by a uniform magnetic field in a region 5 cm long (along the path of the electron). The magnetic field is $6 \times 10^{-4}\,\text{W/m}^2$. Through what angle is the electron deflected? How fast is the electron going? Is the electron's speed affected by the magnetic field? The electron mass is 9.1×10^{-31} kg.

Note 1: If needed, Eq. 2 of this unit may be given to you on the exam. However, the functional form $dN_s/d\Omega = 1/\sin^4(\theta/2)$ should be known to you: if that is all that is required, it will not be given.

Note 2: You will be expected to know the charge on an electron: 1.6×10^{-19} C.

Note 3: In this unit, you may always use non-relativistic formulae.

Brief Answers:

2. $$\frac{\#}{\text{sec}} = (A/\ell^2)nt(d\sigma/d\Omega)N_i$$
$$= (10/900) \cdot 10^{27} \cdot 10^3 \cdot 10^{-26} \cdot 10^5$$
$$= 11.1\,\text{counts/sec.}$$

3. $ma = F$

$$\frac{mv^2}{r} = evB$$

so: $r = \dfrac{mv}{eB}$

$\ell = 5\,\text{cm}$

$$\theta = \frac{\ell}{r} = \frac{\ell eB}{mv}$$

but v is given by kinetic energy due to acceleration

$mv^2/2 = eV$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{(2)(1.6 \times 10^{-19}\,\text{C})(3 \times 10^3\,\text{V})}{9.1 \times 10^{-31}\,\text{kg}}} = 3.25 \times 10^7\,\text{m/s}$$

Therefore: $\theta = \dfrac{\ell eB}{m\sqrt{2eV/m}} = \ell B \dfrac{\sqrt{e/m}}{\sqrt{2V}} = 0.162\,\text{radians.}$

Speed is not affected by the magnetic field because constant-field magnetic force is always perpendicular to the velocity.