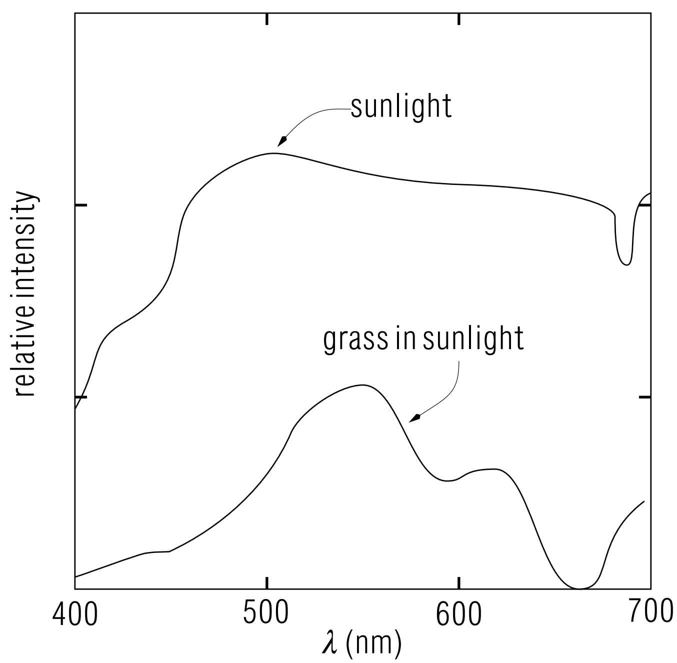
MISN-0-229

COLORS FROM SPECTRAL DISTRIBUTIONS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

COLORS FROM SPECTRAL DISTRIBUTIONS

by

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Title: **Colors from Spectral Distributions**

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Input Skills:

1. Vocabulary: photon, wavelength, nanometer(nm) (MISN-0-212); monochromatic, tristimulus response values (MISN-0-227).
2. Given the wavelength of any monochromatic light beam, state its approximate perceived color (MISN-0-212).
3. Given a mixture of two monochromatic light beams, plus the tristimulus values, determine the relative responses of each of the three types of cone receptors (MISN-0-227).

Output Skills (Knowledge):

K1. Outline the procedure for combining the wavelength distribution of light energy coming from an object and the tristimulus values to produce the light's chromaticity coordinates. Justify the procedure.

Output Skills (Rule Application):

R1. Given the wavelength distribution of light energy coming from an object, and a table of tristimulus values, compute the light's chromaticity coordinates.

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1. Overview

Light entering the human eye normally contains a continuous distribution of wavelengths. The cover of this module shows continuous spectral distributions from the sun (looking directly at it, which you should never do) and from fresh lawn grass in sunlight. These curves show the distribution of energy that would enter your eye in each case. Note that the light entering your eye when you are looking at lawn grass peaks in the green part of the spectrum, as you would expect. In this module we show you how to calculate the chromaticity coordinates for such continuous spectral distributions.

2. Method

2a. Actual-Light Times Tri-Stimulus Responses. Suppose we have the goal of determining the (numerical) perceived color of some light entering the eye. Suppose also that we are given the spectral distribution of that light entering the eye. To obtain the chromaticity coordinates of the light (hence its numerical color) we: (1) chop the eye's tri-stimulus response curves into narrow wavelength bands; (2) multiply the amount of tri-stimulus response in each band by the amount of light actually coming into the eye in that band; (3) sum each of the three responses over all bands; and (4) normalize the responses.

2b. Origin of the Multiplication. Basically, the multiplication of the actual-light distribution by the tri-stimulus values comes about because the tri-stimulus values are what the cone responses would be if the incoming light had a constant spectral distribution (see Fig. 1). That is: the tri-stimulus values are only responses to light with an "even" distribution, meaning one that has a constant spectral distribution (in the case at hand, the same amount of total energy in each band).[1]

[1]Of course one cannot really produce an even-distribution light beam, but some laboratory sources and the equatorial sun at its zenith come close. An even distribution has been chosen as the standard so as to make calculations easier.

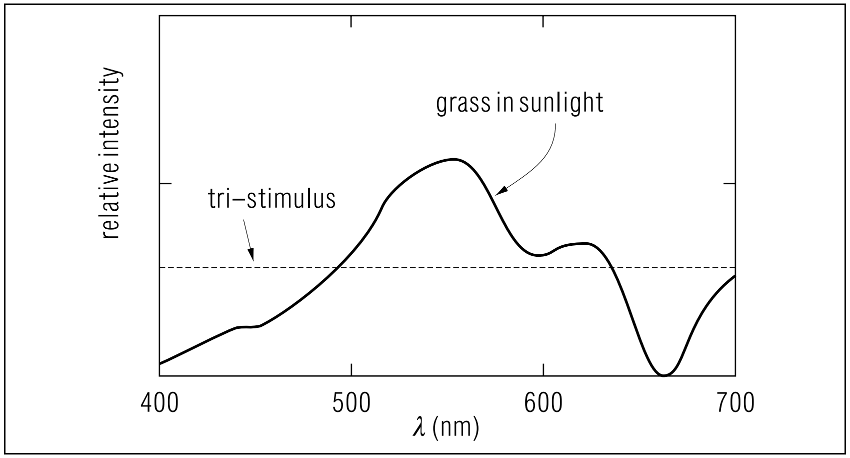

Figure 1. Spectral distribution of light from summer grass and the "even" distribution assumed for the 1931 CIE Tri-Stimulus values.

Of course the light reaching the eye does not, in general, have such an even distribution so the energy in each wavelength band must be corrected for the difference between its actual level and the level it would have in the constant-distribution standard. Assuming linear responses in the eye, this is just a multiplication.

2c. Setting the Incoming-Light Intensity. Now the chromaticity coordinates of a light beam are independent of the over-all intensity of that beam, so we can set the level of the light entering the eye at a value that makes the calculations easiest. Thus we set the *average* level of the incoming light at the constant value assumed for the tri-stimulus ("even") distribution.

2d. Deducing the Multiplication. With the average level of the actual light set at the constant level that produces the tri-stimulus values, it is obvious that the actual-light cone responses in a band are the actual light intensity times the three tri-stimulus values for that band. Thus if in some band the actual light has half the intensity of the same band in the even-distribution light, then that band in the actual light will produce half the cone response of the same band in the even-distribution (tri-stimulus) light. Similarly, an actual-light band with three times as much light as the corresponding even-distribution band produces three times the response of the even-distribution (tri-stimulus) band.

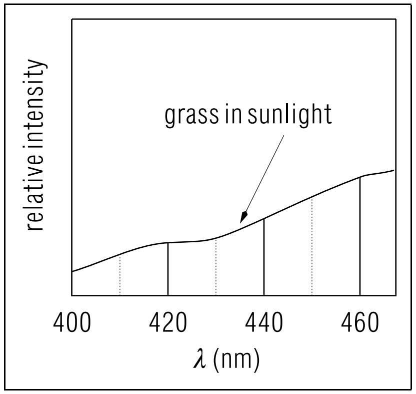

Figure 2. Illustration of breaking the "grass" curve of Fig. 1 into 20 nm-wide bands for the purpose of digitizing the curve.

2e. Getting the Chromaticity Coordinates. Once we have the actual-light responses in each band for each type of cone, we sum the responses for each cone over all the bands. That gives us the total actual-light response for each of the three cone types. From those three numbers, commonly designated X, Y, and Z, we calculate the chromaticity coordinates x and y in the usual way.[2]

3. Practical Details

3a. Digitization of the Continuous Distributions. To approximate a continuous distribution by a complete set of constant-width rectangular bands, we will use the simple method of setting the height of each band at the value of the band's center point. Here we perform this "digitization" for each of the four distributions involved: the spectral distribution of the actual light coming into the eye and the tri-stimulus spectral response curves for each of the eye's three types of cones. For example, suppose we take bands 20 nm wide (e.g. 400-420 nm, 420-440 nm, ... as in Fig. 2) for the "grass in sunlight" curve on the cover of this module with corresponding values in the Appendix table. Then we say that to a good approximation the the amount of light in the 420-440 nm band is the value quoted in the Appendix table for summer grass in sunlight at the band's center point, 430 nm. Similarly, we take the 420-440 nm cone responses to be their values at 430 nm.

There are various ways that professionals make the "digitization" process more accurate, such as by using narrower bands or by piecewise fitting of the curves with functions and then multiplying the functions.

[2]See "Color Specification: Chromaticity," MISN-0-227.

3b. Summary. Here is a summary of the procedure we use:

1. multiply the light intensity at the center of each band by each of the three tri-stimulus values (x_λ, y_λ, z_λ) at the center of that band;
2. sum those products separately for each cone type, producing three integrated response values; X, Y, and Z.

▷ Use the Appendix table to find the integrated response intensities for summer grass in sunlight. We find $X = 6766$, $Y = 10058$, and $Z = 2608$. *Help: [S-1]*

▷ Use the table in the Appendix to find the chromaticity coordinates for human skin illuminated by a 60 Watt light bulb, and summer grass in sunlight. We find $x = 0.51$, $y = 0.40$ for the first, $x = 0.35$, $y = 0.52$ for the second. *Help: [S-2]*

Acknowledgments

I would like to thank Richard Hall, Richard McCoy, T. H. Edwards and Beth Wendt for their very helpful reviews. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

A. Data for Calculations

λ	sg,sl	hs,sb	x_λ	y_λ	z_λ
400	47	23	14	0	68
410	87	30	44	1	207
420	103	43	134	4	646
430	110	55	284	12	1386
440	164	69	348	23	1747
450	201	85	336	38	1772
460	238	97	291	60	1669
470	273	117	195	91	1288
480	308	141	96	139	813
490	448	158	32	208	465
500	644	176	5	323	272
510	941	206	9	503	158
520	1163	227	63	710	78
530	1271	235	166	862	42
540	1366	256	290	954	20
550	1369	294	433	995	9
560	1300	367	595	995	4
570	1131	376	762	952	2
580	853	422	916	870	2
590	615	491	1026	757	1
600	525	609	1062	631	1
610	549	692	1003	503	0
620	559	828	854	381	0
630	470	923	642	265	0
640	294	1022	448	175	0
650	82	1127	284	107	0
660	0	1177	165	61	0
670	151	1227	87	32	0
680	322	1275	47	17	0
690	509	1323	23	8	0
700	975	1369	11	4	0

Note: The second and third columns contain "relative" values: if all numbers in one of those columns are multiplied by any single number, it will have no effect on the column's chromaticity coordinates. All the numbers in the fourth, fifth, and sixth columns, together, can be multiplied by the same single number and it will have no effect on deduced chromaticity coordinates. λ: wavelength in nm.

sg,sl: summer grass in sunlight: see Note above. References are (a) and (d) below.

hs,sb: human skin illuminated by standard 60 Watt "light bulb"; see Note above. References are (b) and (e) below. All human skin is close to being the same color, in the sense of chromaticity coordinates, but the *intensity* of the light coming from skin varies immensely over people that arose (in an evolutionary sense) from different climates (these are differences in skin "reflectivity").

$\mathbf{x}_\lambda$**,** $\mathbf{y}_\lambda$**,** $\mathbf{z}_\lambda$: 1931 CIE Standard Observer Tri-stimulus Values: see Note above. Reference is (c) below. Values shown are actual values × 1000 (e.g. actual value is: $x_\lambda(400\,\text{nm}) = 14/1000 = 0.014$). The use of whole numbers makes calculator or computer entry easier.

References:

a. E. L. Krinov, *Spectral Reflectance Properties of Natural Formations*, translation by G. Belkov, issued as Technical Translation 439, National Research Council of Canada, Ottawa (1953), Appendix I, No. 164.

b. E. A. Edwards and S. Q. Duntley, *Science 90*, 235 (1939), graph digitized by present author.

c. G. Wyszecki and W. S, Stiles, *Color Science*, John Wiley and Sons, Inc., New York (1967), Table 3.2.

d. P. Moon, "Proposed Standard Solar Radiation Curves for Engineering Use," *J. Franklin Institute, 230* 583 (1940). Normal incidence, mean solar distance, sea level, all as quoted in Table 2.1 of Ref. (c).

e. Table 1.6 of Ref. (c).

PROBLEM SUPPLEMENT

▷ This symbol marks problems that are in the text. Those problems are part of this module's problem set: do all of them properly before attempting the problems below.

Note: The problem also occurs on this module's Model Exam.

1. Suppose light coming from a particular manufacturer's sample has this relative energy distribution, specified for 20 nm-wide bins whose midpoints go from 400 nm to 700 nm:

   27, 35, 42, 48, 53, 57, 60, 53, 48, 35, 18, 8, 0, 0, 0, 0.

   Compute the light's chromaticity coordinates, given these two of the three integrated response intensities:

   $$X = 145, Y = 216.$$

λ	1000 times		
(nm)	x_λ	y_λ	z_λ
400	14	0	68
420	134	4	646
440	348	23	1747
460	291	60	1669
480	96	139	813
500	5	323	272
520	63	710	78
540	290	954	20
560	595	995	4
580	916	870	2
600	1062	631	1
620	854	381	0
640	448	175	0
660	165	61	0
680	47	17	0
700	11	4	0

Brief Answers:

1. $Z = 243$; $x = 0.24$, $y = 0.36$.

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from TX-3b)*

$X = (47 \times 0.014) + (87 \times 0.044) + (103 \times 0.134) \ldots = 6766$
If you don't understand this, note: In Appendix A the values shown under x_λ, etc. should be divided by 1000 before being used (see note after the table of numbers in Appendix A).

S-2 *(from TX-3b)*

See [S-1] for summer grass in sunlight.
For human skin in 60 W bulb light:
$X = (23 \times 0.014) + (30 \times 0.044) + \ldots = 5623$
$Z = (23 \times 0.068) + (30 \times 0.207) + \ldots = 1002$
Note: Using 20 nm - wide bands produces $X = 2850$, $Z = 499$.
For converting X, Y, and Z to x and y, see MISN-0-227.

MODEL EXAM

1. See Output Skill K1 in this module's *ID Sheet*.

2. Suppose light coming from a particular manufacturer's sample has this relative energy distribution, specified for 20 nm - wide bins whose midpoints go from 400 nm to 700 nm:

   27, 35, 42, 48, 53, 57, 60, 53, 48, 35, 18, 8, 0, 0, 0, 0.

   Compute the light's chromaticity coordinates, given these two of the three integrated response intensities:

   $$X = 145, Y = 216.$$

λ (nm)	1000 times x_λ	y_λ	z_λ
400	14	0	68
420	134	4	646
440	348	23	1747
460	291	60	1669
480	96	139	813
500	5	323	272
520	63	710	78
540	290	954	20
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580	916	870	2
600	1062	631	1
620	854	381	0
640	448	175	0
660	165	61	0
680	47	17	0
700	11	4	0

Brief Answers:

1. See this module's *text*.

2. See this module's *Problem Supplement*, problem 1.