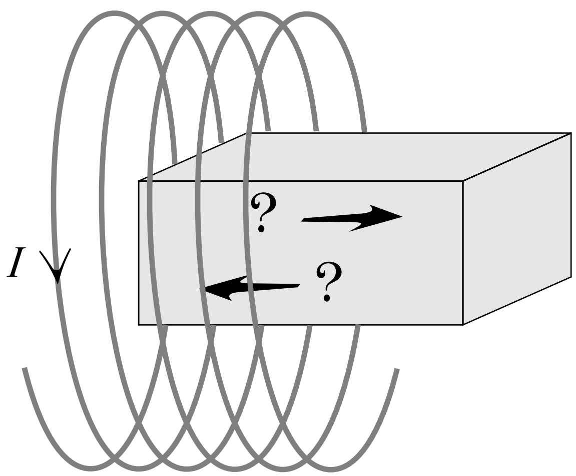
MISN-0-143

STORAGE OF ENERGY IN MAGNETIC FIELDS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

STORAGE OF ENERGY IN MAGNETIC FIELDS

by

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Title: **Storage of Energy in Magnetic Fields**

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Input Skills:

1. Skills in "Magnetic Inductance," (MISN-0-144).

Output Skills (Knowledge):

K1. Derive the expression for the energy stored in the magnetic field associated with a current-carrying circuit, starting from the expression for the self-induced EMF. Explain where this energy comes from, and where it goes when the current is turned off.

K2. Verify for some special cases where the magnetic field is confined to a limited region in space (such as within a toroid or a circuit consisting of coaxial cylindrical sheets of current), that the above expression is equivalent to $B^2/(2\mu_0)$ integrated over the volume where B exists.

Output Skills (Problem Solving):

S1. Given a current configuration, determine the magnetic energy density.

External Resources (Required):

1. M. Alonso and E. J. Finn, *Physics*, Addison-Wesley, Reading (1970); for access see this module's *Local Guide*.

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1. Description

In any current-carrying circuit, the current produces a magnetic field through the loop of the circuit. If a magnetic field through that same loop varies with time, there is induced in that loop an $\mathcal{EMF}$ which drives a current around that loop. It thus follows that if a current in a circuit varies with time (such as when it is first turned on) it produces a time-varying magnetic field which in turn induces an $\mathcal{EMF}$ (with a corresponding current) in this same circuit. This induced $\mathcal{EMF}$ drives a current around the loop in such a direction as to oppose the change associated with the changing current that gave rise to it. Therefore, to merely set up a current requires the expenditure of energy. This energy can be viewed as going to set up the magnetic field that results. This energy is the topic of this module.

2. Suggested Procedure

In AF[1] study section 20.7.

Note that $\epsilon_0 \equiv 1/(4\pi k_e)$ and $\mu_0 \equiv 4\pi k_m$.

Output Skill K1 is the derivation of equation 20.14.

Output Skill K2 is being able to do what's done for the case of coaxial cylindrical sheets in the shaded portion on page 464.

Work problem 20.14.

Answers to Problem:

20.14 Refer to section 17.9 and Example 19.10.

a. The energy density at a point 1 meter from the wire is $(2/\pi) \times 10^{-7}\,\mathrm{J/m^3}$ (be sure to carry along the units to see that the dimensions work out).

[1] M. Alonso and E. J. Finn, *Physics*, Addison-Wesley, Reading (1970). For access, see this module's *Local Guide*.

b. The radius given is the radius of the large ring, not the radius of the cross-section of the doughnut. The answer is $(0.8/\pi)\,\mathrm{J/m^3}$.

LOCAL GUIDE

The readings for this unit are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them as "The readings for CBI Unit 143." Do **not** ask for them by book title.

PROBLEM SUPPLEMENT

Note: Problem 2 also occur in this module's *Model Exam*.

1. a. Write down the expression for the $\mathcal{EMF}$ induced in a circuit when the current varies with time. [H]
   b. In a circuit where the $\mathcal{EMF}$ is V_E what is the instantaneous power that must be supplied by the source of the $\mathcal{EMF}$ when the current at the instant has the value I? [D]
   c. Using the above two results write down the expression for the rate at which energy must be supplied to a circuit to oppose the induced $\mathcal{EMF}$. [B]
   d. Where does this energy go? [K]
   e. What total energy is expended when the current in a circuit builds from zero to value I? [F]

2. Refer to the circuit of the toroidal coil shown in figure 19.35 in AF.
   a. Calculate the self-inductance of the toroidal coil (in Fig. 19.35, L, L', and L" are lengths, not self-inductances. To avoid confusion replace these labels by $\mathcal{L}$, $\mathcal{L}'$, and $\mathcal{L}''$. The outer shaded circle should have been labeled $\mathcal{L}'$). [L]
   b. When the current in the coil is I, what is the value of the energy stored in the magnetic field associated with the coil? [E]
   c. Find the expression for the B-field within the toroid. [A]
   d. What is the energy-density associated with the field within this volume? [J] Is it uniform? [C]
   e. From this density find the total energy in the field inside this coil. [G] Compare it to the answer you got for (b). [M]
   f. Why did you not integrate this expression over a larger region of space, all of space for example? [I]

Brief Answers:

A. $\mu_0 nI$.

B. $LI(dI/dt)$.

C. Yes.

D. $V_E I$.

E. $\frac{1}{2}\mu_0 n^2 I^2 \pi R^2 \mathcal{L}$.

F. $\frac{1}{2}LI^2$.

G. $\frac{1}{2}\mu_0 n^2 I^2$ times the volume inside the doughnut ($\pi R^2 \mathcal{L}$ if $\mathcal{L} \gg R$).

H. $L(dI/dt)$ where L is the self-inductance of the circuit.

I. B is zero outside the doughnut region.

J. $\frac{1}{2}\mu_0 n^2 I^2$.

K. Into the magnetic field established by the current.

L. $\mu_0 n^2 \pi R^2 \mathcal{L}$, n is number of turns per unit length, R is the radius of the cross-section, $\mathcal{L}$ is the length around the center line of the toroid.

M. It is the same.

MODEL EXAM

1. See Output Skills K1-K2 in this module's *ID Sheet*. The actual exam may have one or both of these skills, or neither.

2. Refer to the circuit of the toroidal coil shown in figure 19.35 in AF.

   a. Calculate the self-inductance of the toroidal coil (in Fig. 19.35, L, L', and L" are lengths, not self-inductances. To avoid confusion replace these labels by $\mathcal{L}$, $\mathcal{L}'$, and $\mathcal{L}''$. The outer shaded circle should have been labeled $\mathcal{L}'$).

   b. When the current in the coil is I, what is the value of the energy stored in the magnetic field associated with the coil?

   c. Find the expression for the B-field within the toroid.

   d. What is the energy-density associated with the field within this volume? Is it uniform?

   e. From this density find the total energy in the field inside this coil. Compare it to the answer you got for (b).

   f. Why did you not integrate this expression over a larger region of space, all of space for example?

Brief Answers:

1. See this module's *text*.

2. See this module's *Problem Supplement*, problem 2.